ADVANCED PLACEMENT PHYSICS MECHANICS TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS
Universal gravitational constant, $G = 6.67 \times 10^{-11}\ \text{m}^3/(\text{kg}\cdot\text{s}^2) = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$
Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Magnitude of the gravitational field strength at the Earth's surface, $g = 9.8\ \text{N/kg}$

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p

UNIT SYMBOLS				
	hertz,	Hz	newton,	N
	joule,	J	second,	s
	kilogram,	kg	watt,	W
	meter,	m		

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

- The frame of reference of any problem is assumed to be inertial unless otherwise stated.
- Air resistance is assumed to be negligible unless otherwise stated.
- Springs and strings are assumed to be ideal unless otherwise stated.

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MECHANICS

Unit 1 Kinematics

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x\left(x - x_0\right)$$

$$\Delta x = \int v_x(t)\,dt$$

$$\Delta v_x = \int a_x(t)\,dt$$

Unit 2 Force & Translational dynamics

$$\vec{x}_{\text{cm}} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$\vec{r}_{\text{cm}} = \frac{\int \vec{r}\,dm}{\int dm}$$

$$\lambda = \frac{d}{d\ell}m(\ell)$$

$$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}} = \frac{\vec{F}_{\text{net}}}{m_{\text{sys}}}$$

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$

$$\left|\vec{F}_f\right| \le \left|\mu \vec{F}_N\right|$$

$$\vec{F}_s = -k\Delta\vec{x}$$

$$a_c = \frac{v^2}{r} = r\omega^2$$

$$T = \frac{1}{f}$$

Unit 3 Work Energy and Power

$$K = \frac{1}{2}mv^2$$

$$W = \int_a^b \vec{F}\cdot d\vec{r}$$

$$\Delta K = \sum W_i = \sum F_{\|,i}d_i$$

$$\Delta U = -\int_a^b \vec{F}_{\text{cf}}(r)\cdot d\vec{r}$$

$$F_x = -\frac{dU(x)}{dx}$$

$$U_s = \frac{1}{2}k\left(\Delta x\right)^2$$

$$U_G = -G\frac{m_1 m_2}{r}$$

$$\Delta U_g = mg\Delta y$$

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$P_{\text{inst}} = \frac{dW}{dt}$$

Unit 4 linear momentum

$$\vec{p} = m\vec{v}$$

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t)\,dt = \Delta\vec{p}$$

$$\vec{v}_{\text{cm}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

a = acceleration
E = energy
f = frequency
F = force
h = height
J = impulse
k = spring constant
K = kinetic energy
ℓ = length
m = mass
M = mass
p = momentum
P = power
r = radius, distance, or position
t = time
T = period
U = potential energy
v = velocity or speed
W = work
x = position or distance
y = height
λ = linear mass density
μ = coefficient of friction

Unit 5 Torque and Rotational Dynamics

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\left(\theta - \theta_0\right)$$

$$v = r\omega$$

$$a_T = r\alpha$$

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$I_{\text{tot}} = \sum I_i = \sum m_i r_i^2$$

$$I = \int r^2\,dm$$

$$I' = I_{\text{cm}} + Md^2$$

$$\alpha_{\text{sys}} = \frac{\Sigma\tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$$

Unit 6 Energy and momentum of Rotating Systems

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$W = \int \tau\cdot d\theta$$

$$\vec{L} = \vec{r}\times\vec{p} = I\vec{\omega}$$

$$\Delta L = \int \tau\,dt$$

$$\Delta x_{\text{cm}} = r\Delta\theta$$

Unit 7 Oscillations

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T_{\text{phys}} = 2\pi\sqrt{\frac{I}{mgd}}$$

$$x = x_{\text{max}}\cos\left(\omega t + \phi\right)$$

a = acceleration
d = distance
f = frequency
F = force
I = rotational inertia
k = spring constant
K = kinetic energy
ℓ = length
L = angular momentum
m = mass
M = mass
p = momentum
r = radius, distance, or position
t = time
T = period
v = velocity or speed
W = work
x = position or distance
α = angular acceleration
θ = angle
τ = torque
ϕ = phase angle
ω = angular frequency or angular speed

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GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

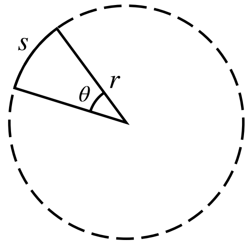
$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$



A = area
b = base
C = circumference
h = height
ℓ = length
r = radius
s = arc length
S = surface area
V = volume
w = width
θ = angle

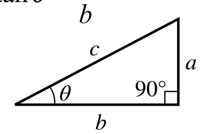
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



VECTORS

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

$\vec{r} = \left(A\hat{i} + B\hat{j} + C\hat{k}\right)$

$\vec{C} = \vec{A} + \vec{B}$

$\vec{C} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$

CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\ln ax\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left[\sin(ax)\right] = a\cos(ax)$$

$$\frac{d}{dx}\left[\cos(ax)\right] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

IDENTITIES

$\log\left(a\cdot b^x\right) = \log a + x\log b$

$\sin^2\theta + \cos^2\theta = 1$

$\sin\left(2\theta\right) = 2\sin\theta\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

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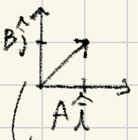
Unit 1, Kimenatics

Scalars = quantities
Vectors = quantities w magnitude

Vectors: Velocity, acceleration, displacement
Scalars: speed, speed (basically no sign), Distance traveled

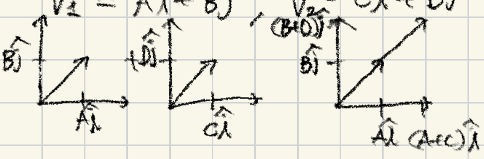
Can be written in Vector notation / arrows.



or $A\hat{i} + B\hat{j}$

Can be calculated accordingly

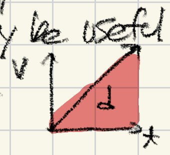
$\vec{V_1} = A\hat{i} + B\hat{j}$, $\vec{V_2} = C\hat{i} + D\hat{j} \rightarrow \vec{V_1} + \vec{V_2} = (A+C)\hat{i} + (B+D)\hat{j}$



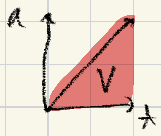
$\Delta x = x - x_0 \rightarrow$ Displacement. change in position
Doing stuff w displacement & time = Kinematics
$v =$ displacement$/dt$, $a =$ displacement$/dt^2$
Displacement $\xrightarrow{d/dt}$ Velocity $\xrightarrow{d/dt}$ acceleration
$\rightarrow$ Calculous ??

$V_x = V_0 + a_x t \rightarrow$ Velocity in constant acceleration
$x = x_0 + V_0 t + \frac{1}{2} a t^2 \rightarrow$ displacement in "
$V_x^2 = V_0^2 + 2a(x - x_0) \longrightarrow$ May be useful in weird problems.

$\Delta x = \int_{t_1}^{t_2} V_x(t)\,dt \longrightarrow$

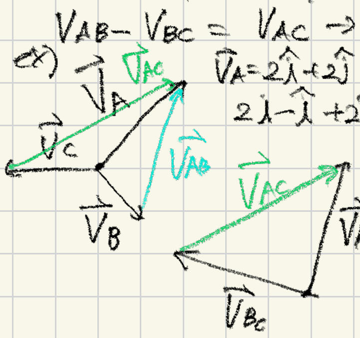


$\Delta V = \int_{t_1}^{t_2} a_x(t)\,dt \rightarrow$



Reference frames will determine the direction & magnitude of Vectors
V_{AB} Velocity of A when B is looking
$= V_A - V_B$ (Velocity of A for stationary $-$ V of B for stationary)
$V_{AB} - V_{BC} = V_{AC} \rightarrow$ You can find relative velocity implicitly

ex)



$\vec{V_A} = 2\hat{i} + 2\hat{j}$, $\vec{V_B} = +\hat{i} - \hat{j}$
$2\hat{i} - \hat{i} + 2\hat{j} + \hat{j} = \hat{i} + 3\hat{j} \rightarrow$ makes sense

$\vec{V_{AB}} - \vec{V_{BC}} = \hat{i} + 3\hat{j} - (-3\hat{i} + \hat{j})$
$4\hat{i} + 2\hat{j} = \vec{V_{AC}}$

equations

$V_x = V_0 + at$
$x = x_0 + V_0 t + \frac{1}{2} a t^2$
$V^2 = V_0^2 + 2a(x - x_0)$
$\Delta x = \int V_x(t)\,dt$
$\Delta V_x = \int a_x(t)\,dt$
$V_A - V_B = V_{AB}$
$V_{AB} - V_{BC} = V_{AC}$

Unit 2 Force & Translational motion

We can look at a system as its center of mass.
* Read exam description

The most general equation for Center of mass:

$r_{cm} = \frac{1}{M}\int_0^{r_i} r\,dm \rightarrow dm =$ infinitesimal mass
$r =$ any variable in which mass is distributed (could be x, y, z, area, volume, etc...)

how?
$r_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{\int_0^{r_i} dm\,\vec{r}_i}{\int_0^{r_i} dm} \rightarrow \int_0^{r_i} dm = M \rightarrow \frac{1}{M}\int_0^{r_i} dm\,\vec{r}_i$

$\rightarrow \lambda = \frac{\text{total mass}}{\text{total } r} \rightarrow$ can be whatever. $= \frac{dm}{dr}$

$dr\,\lambda = dm$
$\rightarrow r_{cm} = \frac{1}{M}\int_0^{r_i} r \times dr \times \lambda$

Shortcuts

For objects/particles along a single axis:

$$y_{cm} = \frac{\sum m\vec{x}}{\sum m_i}$$

For uniform density solid objects along 1 axis:

$\frac{1}{M}\int_0^{x} x \cdot dx \cdot \lambda = x_{cm}$
$\rightarrow \frac{\lambda}{m}\int_0^{x} x\,dx \rightarrow \frac{\lambda}{M}\left(\frac{1}{2}x^2\right)\Big|_{x=0}^{x=l} \rightarrow \left(\frac{\lambda}{M}\right)\left(\frac{l^2}{2}\right) \rightarrow M = \lambda l \rightarrow \frac{\lambda l^2}{2(\lambda l)} = \frac{l}{2}$

For nonuniform density solid objects along 1 axis

$\frac{1}{M}\int_0^{x} x\,dx\,\lambda = \frac{\lambda_0}{M}\int_0^{M} x\left(1+\frac{x}{l}\right)dx = \frac{\lambda_0}{M}\left[\frac{5l^2}{6}\right] \rightarrow$ to make it in respect to l, $M = \int_0^{l} dm \rightarrow M = \int_0^{l} \lambda\,dx \rightarrow M = \int_0^{l} \lambda_0\left(1+\frac{x}{l}\right)dx$

$\rightarrow M = \lambda_0\left[\frac{3l}{2}\right], \quad \frac{\lambda_0}{M}\left[\frac{5l^2}{6}\right] \rightarrow \frac{2}{3l}\left[\frac{5l^2}{6}\right] = \frac{5}{6}l$

equations

Same thing
$x_{cm} = \frac{\sum mx}{\sum m}$
$r_{cm} = \frac{\int \vec{r}\,dm}{\int dm}$

$\lambda = \frac{d}{dl}m(l)$

$\vec{a}_{sys} = \frac{\sum \vec{F}}{m_{sys}} = \frac{\vec{F}_{net}}{m_{sys}}$

$|\vec{F}_g| = G\frac{m_1 m_2}{r^2}$

$|\vec{F}_f| \leq |\mu \vec{F}_N|$

$\vec{F}_s = -k\Delta\vec{x}$

$a_c = \frac{v^2}{r} = r\omega^2$

$T = \frac{1}{f}$

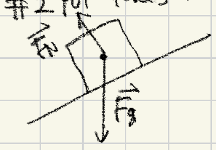
$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A} \rightarrow$ for all forces exerted by an object to another object, there is a force equal in magnitude & opposite in direction.

☆ $\sum F = ma \rightarrow$ acceleration of a system's CM × Mass of system = Net force.
↳ if $\sum F = 0$, $a = 0 \rightarrow V$ is either Constant / zero.

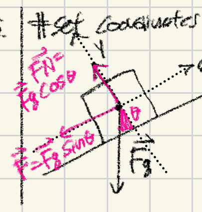
Free Body diagrams ☆☆

#1 find C.M

#2 put forces in as arrows.



set coordinates as you please (usually at the direction of acceleration)

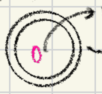


→ Forces ✓✓✓

$F_{gravity} = G\frac{m_1 m_2}{r^2} \rightarrow F_{gravity}/m_1 = G\frac{m_2}{r^2} = \vec{g}$. $\vec{g}$ is usually $9.8\,m/s^2$ on Earth's Surface

→ we consider the cm gravity as a sum of individual forces from differential masses.

Shell theorem:

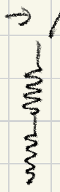


→ net gravitational force = 0
→ do it normally.

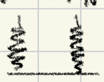
→ gravitational force of $\frac{4}{5}R$ radius sphere $\frac{4}{5}R$ away from center. $\left(m_{partial} = \rho\frac{4}{3}\pi(r_{partial})^3\right.$ ↳ density

$F_{friction} = \mu \times F_N \rightarrow \mu_s F_N$ (static) $\mu_k F_N$ (kinetic)

$F_{spring} = -k\Delta x \rightarrow$



$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$
equivalent k
< smallest k

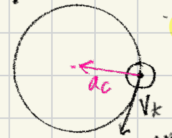


$k_{eq} = k_1 + k_2$

$F_{resistive}$ ☆☆☆ $= -kv \rightarrow$ velocity dependant, leads to a differential equation for velocity
$F = ma$, $F = m\frac{dv}{dt} \rightarrow m\frac{dv}{dt} = mg - kv \rightarrow v(t) = \frac{mg}{k}\left(1 - e^{-\frac{k}{m}t}\right)$

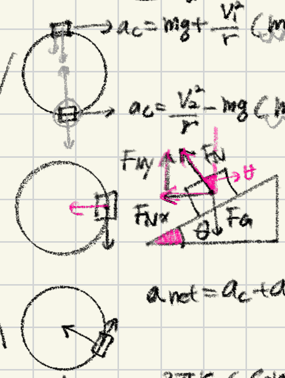
Circular motion

$a_c = \frac{v^2}{r} \rightarrow$ it directs to the center
centripetal a causes tangential V.



The a_c constantly changes the direction of the V_t, making the velocity change in direction not magnitude

examples



$\rightarrow a_c = mg + \frac{v_1^2}{r}$ (minimum speed) $[v = \sqrt{gr} \rightarrow$ when only g is $a_c]$

$\rightarrow a_c = \frac{v_2^2}{r} - mg$ (maximum speed)

$a_c = F_N\sin\theta \leftrightarrow \frac{g}{\cos\theta} \times \sin\theta = a_c$
$= \tan\theta\, g$

$F_N\cos\theta = mg$
$F_N = mg/\cos\theta$

$a_{net} = a_c + a_T$ (vector sum)

$T = \frac{1}{f}$, $T = \frac{2\pi r}{v}$ (constant speed)

☆ satelite!!!
$T^2 = \frac{4\pi^2}{GM}R^3 \rightarrow a_c$ is by gravity.

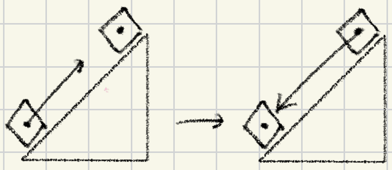
Unit 3 Work Energy and Power

Energy: the ability to do work → cannot be negative (scalar)

Translational $KE = \frac{1}{2}mv^2$
→ KE is different for spotters because the percieved v is different.

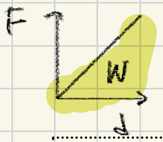
Work = energy put out/into a system by a force over a distance.
$W = F \cdot$ (distance or displacement)

Conservative Work: depends on displacement
→ F_g, F_s, etc
nonconservative work: depends on path/distance traveled.
→ F_f, F_{drag}, etc.

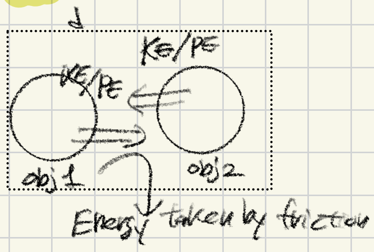


Conservative
work: 0
(because same x, y)
nonconservative
work > 0
(because distance is added for friction)

force only of the direction of displacement
$\int_a^b F \cdot \cos\theta \cdot d\vec{r}$, $F\cos\theta d = W$
$\Delta K = \sum F \cdot d$ → change in kinetic energy = sum of net work.



area under $F/d = W$



→ within a system, conservative forces can change KE → PE and vice versa.
→ However, the nonconservative forces may take in/out energy of the system.

Potential energy: scalar quantity associated with the position of objects within a system.

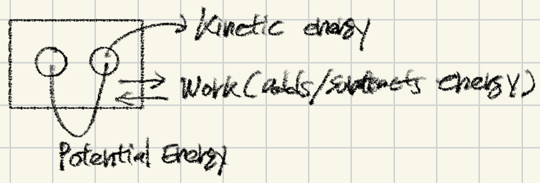
$$\Delta PE = -\int_a^b F(r)\cos\theta \, d\vec{r} = W$$

☆ different types of potential energy.

$$U_{spring} = \frac{1}{2}k(\Delta x)^2$$

$$U_{gravity} = G\frac{m_1 m_2}{r^2} \;/\; \Delta U_g = mg\Delta y$$

Potential Energy & Kinetic Energy & Work.



$E_{mech} = K_E + P_E$
any work done in/out of a system ⇒ ΔE_{mech}

$$Power = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = \frac{dW}{dt}$$

equations

$K = \frac{1}{2}mv^2$
$W = \int_a^b \vec{F} \cdot d\vec{r}$
$\Delta K = \sum W_i = \sum F_{i,i} d_i$
$\Delta U = -\int_a^b \vec{F}_{cf}(r) \cdot d\vec{r}$
$F_x = -\frac{dU}{dx}$
$U_s = \frac{1}{2}k(\Delta x)^2$
$U_G = -G\frac{m_1 m_2}{r}$
$\Delta U_g = mg\Delta y$
$P_{avg} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$
$P_{int} = \frac{dW}{dt}$

Unit 4 momentum

An object at rest remains at rest & an object in motion remains in motion.

$\vec{p} = m\vec{v}$ → linear momentum ; shows the tendency of an object to remain in motion

Colisions: Force between objects >>> Net external force of the objects/system.

explosion: Forces internal to the system move objects in the system apart

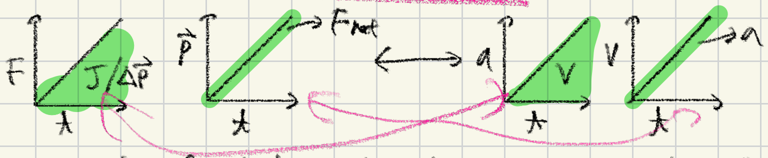
→ we only analyze the final & initial states of the system.

Impulse : $\Delta\vec{p} = J$

→ $\vec{p} = m\vec{v}$, $d\vec{P}/dt = m\vec{v}/dt$ → $d\vec{p}/dt = F_{net}$

→ $\int_a^b (d\vec{P}/dt = F_{net})$ → $\vec{P}_b - \vec{P}_a = \int_a^b F_{net}\,dt$ → $\vec{J} = \int_a^b F(t)\,dt = \Delta\vec{p}$

∴ $\vec{F} = \frac{d\vec{p}}{dt}$ $\int \vec{F}\,dt = \Delta\vec{p} = \vec{J}$



→ only difference is ×m.

a system of objects can be described as one system with one cm velocity.

$$\vec{V}_{cm} = \frac{\sum \vec{P}_i}{\sum m_i} = \frac{\sum m_i\vec{v}_i}{\sum m_i}$$ → constant in the absence of a net external force

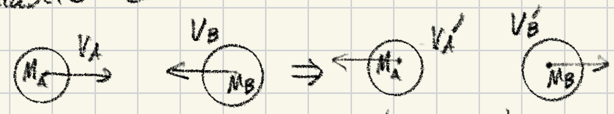
Conservation of momentum: any change to the momentum of an object within a system must be balanced out by an opposite change in momentum elsewhere in the system.

→ this includes colisions (both elastic & inelastic)

Elastic Colisions: both momentum and kinetic energy is conserved.

Inelastic colisions: momentum is conserved.

Elastic Colision

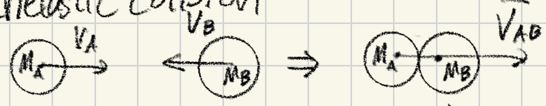


$m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$

Because Kinetic E is conserved:

$V_A + V_A' = V_B + V_B'$

Inelastic Colision



$m_A V_A + m_B V_B = (m_A + m_B)\vec{V}_{AB}$

→ some/all of the initial KE is lost as heat/other forms of energy

※ when two objects of equal mass collide,

Elastic: the objects exchange velocities

Inelastic: the objects have V_{AB} of $(V_A + V_B)$

equations

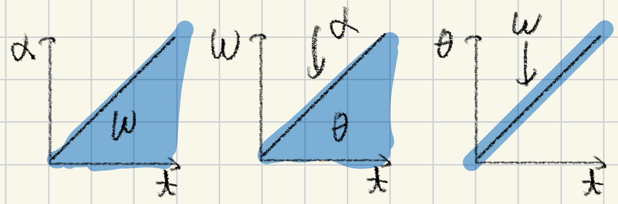
$\vec{p} = m\vec{v}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int_{t_2}^{t_1} F_{net}(t)\,dt = \Delta\vec{p}$

$\vec{V}_{cm} = \frac{\sum \vec{P}_i}{\sum m_i} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$

Unit 5 Torque & Rotational Dynamics.



Rotation: circular movement of an object around an axis or center
→ circular motion of all of the particles in the object.

ω = angular velocity
α = angular acceleration
θ = angular displacement

$\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$, $\alpha t + \omega_0 = \omega$, $\theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = \theta$, $\omega_0^2 + 2\alpha(\theta - \theta_0) = \omega^2$

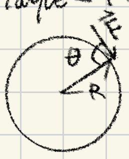
same Kinematic relationship.

$\theta \times R = s$
→ linear displacement = Radius × angular displacement

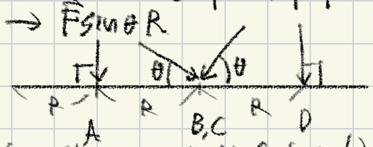
- $R \cdot \theta = d$
- $R \cdot \omega = v$
- $R \cdot \alpha = a$

Not center of mass ☺

Torque = F × distance it is exerted from the AXIS OF ROTATION!!! (equivalent of force in linear motion)



※ The force component perpendicular to the axis of rotation is all that matters.
→ $\vec{F}\sin\theta R$



A: $\vec{F} \times \sin 90 \times R$
B: $\vec{F}\sin\theta \times 2R$
C: $-\vec{F}\sin\theta \times 2R$
D: $\vec{F}\sin 90 \times 3R$

Rotational Inertia = mass (infinitesimal) × distance from the axis of rotation² (equivalent of mass in linear motion)

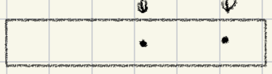
→ $dm \times dr \times dr$ = I of 1 particle

$\int_0^r r^2 dm = I$

$I' = I_{cm} + Md^2$

derivation (if you even care...):

$r' = r + d \rightarrow I = \int (r+d)^2 dm$
mass from CM; mass from the new axis
$\rightarrow \int (r^2 + 2rd + d^2) dm = \int r^2 dm + 2d \int r dm + d^2 \int dm$
$\rightarrow I_{cm} + \underbrace{2d \int r dm}_{0} + \underbrace{d^2 \int dm}_{M}$
$\rightarrow I' = I_{cm} + Md^2$



Rotational equilibrium = $\sum \tau = 0$
→ α will be nonzero, ω will change.
→ This broken when: I changes, τ changes (anything that disturbs α)
↳ $\alpha = \frac{\tau}{I}$ $\left(a = \frac{F}{m}\right)$

equations

$\omega = \frac{d\theta}{dt}$

$\alpha = \frac{d\omega}{dt}$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$v = r\omega$

$a_T = r\alpha$

$\vec{\tau} = \vec{r} \times \vec{F}$

$I = \sum I_i = \sum m r_i^2$

$I = \int r^2 dm$

$I' = I_{cm} + Md^2$

$\alpha = \frac{\sum \tau}{I_{sys}} = \frac{\tau_{net}}{I_{sys}}$