2020

AP® CollegeBoard

Please note: Some of the questions in this former practice exam may no longer perfectly align with the AP exam. Even though these questions do not fully represent the 2020 exam, teachers indicate that imperfectly aligned questions still provide instructional value. Teachers can consult the Question Bank to determine the degree to which these questions align to the 2020 Exam.

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From the 2019 Administration

AP® CollegeBoard

AP® Physics C: Mechanics

Practice Exam

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Contents

Note: This publication shows the page numbers that appeared in the ***2018–19 AP Exam Instructions*** book and in the actual exam. This publication was not repaginated to begin with page 1.

AP Physics C: Mechanics Exam

Regularly Scheduled Exam Date: Monday afternoon, May 13, 2019

Late-Testing Exam Date: Wednesday morning, May 22, 2019

Section I	**Total Time:** 45 minutes Calculator Allowed **Number of Questions:** 35 *(The number of questions may vary slightly depending on the form of the exam.)* **Percent of Total Score:** 50% **Writing Instrument:** Pencil required
Section II	**Total Time:** 45 minutes Calculator Allowed **Number of Questions:** 3 **Percent of Total Score:** 50% **Writing Instrument:** Pen with black or dark blue ink, or pencil

Before Distributing Exams: Check that the title on all exam covers is ***Physics C: Mechanics***. If there are any exam booklets with a different title, contact the AP coordinator immediately.

What Proctors Need to Bring to This Exam

- ☐ Exam packets
- ☐ Answer sheets
- ☐ AP Student Packs
- ☐ *2018-19 AP Coordinator's Manual*
- ☐ This book—*2018-19 AP Exam Instructions*
- ☐ AP Exam Seating Chart template
- ☐ School Code and Homeschool/Self-Study Codes
- ☐ Extra calculators
- ☐ Extra rulers or straightedges
- ☐ Pencil sharpener
- ☐ Container for students' electronic devices (if needed)
- ☐ Extra No. 2 pencils with erasers
- ☐ Extra pens with black or dark blue ink
- ☐ Extra paper
- ☐ Stapler
- ☐ Watch
- ☐ Signs for the door to the testing room
  - "Exam in Progress"
  - "Phones of any kind are prohibited during the test administration, including breaks"

Students are permitted to use rulers, straightedges, and four-function, scientific, or graphing calculators for this entire exam (Sections I and II). Before starting the exam administration, make sure each student has an appropriate calculator, and any student with a graphing calculator has a model from the approved list on page 53 of the *2018-19 AP Coordinator's Manual*. See pages 50–53 of the *AP Coordinator's Manual* for more information. If a student does not have an appropriate calculator or has a graphing calculator not on the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, they must hand copy, date, and sign the release statement on page 52 of the *AP Coordinator's Manual*.

During the administration of Section II, students may have no more than two calculators on their desks. Calculators may not be shared. Calculator memories do not need to be cleared before or after the exam. Students with Hewlett-Packard 48–50 Series and Casio FX-9860 graphing calculators may use cards designed for use with these calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. **Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use the calculator to remove exam questions and/or answers from the room may result in the cancellation of AP Exam scores.**

Tables containing equations commonly used in physics are included in each AP Exam booklet, for use during the entire exam. Students are NOT allowed to bring their own copies of the equation tables to the exam room.

Students may take both Physics C exams, Mechanics only, or Electricity and Magnetism only. The Mechanics exam is administered first, after which students taking both exams are given a break. Then the Electricity and Magnetism exam is administered. Prior to testing day, determine which exams students are taking. Those taking both Physics C exams and those taking Physics C: Mechanics only should report for the 12 noon start time (11 a.m. in Alaska). Those taking Electricity and Magnetism only should report to the testing room after the break (approximately 2 p.m., 1 p.m. in Alaska). If all students are taking Electricity and Magnetism only, you must not begin the exam before 2 p.m.

The two exams are in separate exam packets, and require separate answer sheets. At the beginning of the session, you will distribute **only** the packets and answer sheets for Mechanics. The materials for Electricity and Magnetism will be distributed after the break.

SECTION I: Multiple Choice

› **Do not begin the exam instructions below until you have completed the appropriate General Instructions for your group.**

This exam includes survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

Make sure that you begin the exam at the designated time. Remember, you must complete a seating chart for this exam. See pages 295–296 for a seating chart template and instructions. See the *2018-19 AP Coordinator's Manual* for exam seating requirements (pages 56–59).

If you are giving the regularly scheduled exam, say:
It is Monday afternoon, May 13, and you will be taking the AP Physics C: Mechanics Exam.

If you are giving the alternate exam for late testing, say:
It is Wednesday morning, May 22, and you will be taking the AP Physics C: Mechanics Exam.

Look at your exam packet and confirm that the exam title is "AP Physics C: Mechanics." Raise your hand if your exam packet contains any title other than "AP Physics C: Mechanics," and I will help you.

Once you confirm that all students have the correct exams, say:

In a moment, you will open the exam packet. By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the *2018-19 Bulletin for AP Students and Parents*.

You may now remove the shrinkwrap from the outside only of your exam packet. Do not open the Section I booklet; do not remove the shrinkwrap from the Section II materials. Put the white seals and the shrinkwrapped Section II booklet aside. . . .

Carefully remove the AP Exam label found near the top left of your exam booklet cover. Place it on page 1 of your answer sheet on the light blue box near the top right corner that reads "AP Exam Label.". . .

If students accidentally place the exam label in the space for the number label or vice versa, advise them to leave the labels in place. They should not try to remove the label; their exam can still be processed correctly.

Listen carefully to all my instructions. I will give you time to complete each step. Please look up after completing each step. Raise your hand if you have any questions.

Give students enough time to complete each step. Don't move on until all students are ready.

Read the statements on the front cover of the Section I booklet. . . .

Sign your name and write today's date. . . .

Now print your full legal name where indicated. . . .

Turn to the back cover of your exam booklet and read it completely. . . .

Give students a few minutes to read the entire cover.

Are there any questions? . . .

You will now take the multiple-choice portion of the exam. You should have in front of you the multiple-choice booklet and your answer sheet. You may never discuss the multiple-choice exam content at any time in any form with anyone, including your teacher and other students. If you disclose the multiple-choice exam content through any means, your AP Exam score will be canceled.

Open your answer sheet to page 2. You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses beginning on page 2 of your answer sheet, one response per question. Completely fill in the circles. If you need to erase, do so carefully and completely. No credit will be given for anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work. Rulers, straightedges, and calculators may be used for the entire exam. You may place these items on your desk. Are there any questions? . . .

You have 45 minutes for this section. Open your Section I booklet and begin.

Note Start Time ____________. Note Stop Time ____________.

Check that students are marking their answers in pencil on their answer sheets and that they have not opened their shrinkwrapped Section II booklets. You should also make sure that Hewlett-Packard calculators' infrared ports are not facing each other and that students are not sharing calculators.

After 35 minutes, say:

> **There are 10 minutes remaining.**

After 10 minutes, say:

> **Stop working and turn to the last page in your booklet. . . .**
>
> **You have 2 minutes to answer Questions 101–106. These are survey questions and will not affect your score. You may not go back to work on any of the exam questions. You may now begin.**

To help you and your proctors make sure students are not working on the exam questions, the two pages with the survey questions are identified with a large S on the upper corner of each page. Give students 2 minutes to answer the survey questions.

Then say:

> **Close your booklet and put your answer sheet on your desk, faceup. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. Sit quietly while I collect your answer sheets.**

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label.

After all answer sheets have been collected, say:

> **Now you must seal your exam booklet using the white seals you set aside earlier. Remove the white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, faceup. I will now collect your Section I booklet. . . .**

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

SECTION II: Free Response

When all Section I materials have been collected and accounted for, say:

> **May I have everyone's attention? Place your Student Pack on your desk. . . .**
>
> **You may now remove the shrinkwrap from the Section II packet, but do not open the exam booklet until you are told to do so. . . .**
>
> **Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .**
>
> **Now take an AP number label from your Student Pack and place it on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .**
>
> **Read the last statement. . . .**
>
> **Using your pen, print the first, middle, and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .**
>
> **Now turn to the back cover. Using your pen, complete Items 1 through 3 under "Important Identification Information." . . .**
>
> **Read Item 4. . . .**

> **I need to collect the Student Pack from anyone who will be taking another AP Exam. Keep it, however, if you will be taking the Physics C: Electricity and Magnetism exam this afternoon. If you have no other AP Exams to take, place your Student Pack under your chair now. . . .**
>
> **Read the information on the back cover of the exam booklet. Do not open the booklet until you are told to do so. Look up when you have finished. . . .**

Collect the Student Packs from students who are taking any other AP Exams this year.

Then say:

> **Are there any questions? . . .**
>
> **Rulers, straightedges, and calculators may be used for Section II. Be sure these items are on your desk. . . .**
>
> **You have 45 minutes to complete Section II. You are responsible for pacing yourself and may proceed freely from one question to the next. You must write your answers in the exam booklet using a pen with black or dark blue ink or a No. 2 pencil. If you use a pencil, be sure that your writing is dark enough to be easily read. If you need more paper to complete your responses, raise your hand. At the top of each extra sheet of paper you use, write only:**
>
> - **your AP number,**
> - **the exam title, and**
> - **the question number you are working on.**
>
> **Do not write your name. Are there any questions? . . .**
>
> **You may begin.**

Note Start Time ______________. Note Stop Time ______________.

You should also make sure that Hewlett-Packard calculators' infrared ports are not facing each other and that students are not sharing calculators.

After 35 minutes, say:

> **There are 10 minutes remaining.**

After 10 minutes, say:

> **Stop working and close your exam booklet. Place it on your desk, faceup. . . .**

If any students used extra paper for a question in the free-response section, have those students staple the extra sheet(s) to the first page corresponding to that question in their free-response exam booklets. Complete an Incident Report after the exam and return these free-response booklets with the extra sheets attached in the Incident Report return envelope (see page 68 of the *2018-19 AP Coordinator's Manual* for complete details).

Then say:

> **Remain in your seat, without talking, while the exam materials are collected. . . .**

Collect a Section II booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box and printed their initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students who are taking Mechanics only any electronic devices you may have collected before the start of the exam.

If you are giving the regularly scheduled exam, say:

You may not discuss or share the free-response exam content with anyone unless it is released on the College Board website in about two days. Your AP Exam score results will be available online in July.

If you are giving the alternate exam for late testing, say:

None of the content in this exam may ever be discussed or shared in any way at any time. Your AP Exam score results will be available online in July.

If any students completed the AP number card at the beginning of this exam and are about to be dismissed say:

Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.

If no students are taking Physics C: Electricity and Magnetism, say:

You are now dismissed.

If some students are taking Physics C: Electricity and Magnetism, say:

Those of you taking Mechanics only are now dismissed.

The students taking the Electricity and Magnetism exam now get a 10-minute break. Remember that the Electricity and Magnetism exam cannot begin before 2 p.m., but should start before 3 p.m.

After the students taking Mechanics only have left, say:

If you will also be taking the Physics C: Electricity and Magnetism exam, please listen carefully to these instructions before we take a 10-minute break. Please put all of your calculators under your chair. Your calculators and all items you placed under your chair at the beginning of this exam, including your Student Pack, must stay there, and you are not permitted to open or access them in any way. You are not allowed to consult teachers, other students, notes, textbooks, or any other resources during the break. You may not make phone calls, send text messages, check email, use a social networking site, or access any electronic or communication device. You may not leave the designated break area. If you do not follow these rules, your score will be canceled. Are there any questions? . . .

You may begin your break. Testing will resume at ______________ .

If you will be administering Physics C: Electricity and Magnetism at 2 p.m., be sure all exam materials are kept secure during the break. When the students return from break, turn to page 237 and begin the exam administration for Physics C: Electricity and Magnetism.

If you have no students taking Physics C: Electricity and Magnetism, return all exam materials to secure storage until they are shipped back to the AP Program. (See page 27 of the *2018-19 AP Coordinator's Manual* for more information about secure storage.)

After-Exam Tasks

Be sure to give the completed seating chart to the AP coordinator. Schools must retain seating charts for at least six months (unless the state or district requires that they be retained for a longer period of time). Schools should not return any seating charts in their exam shipments unless they are required as part of an Incident Report.

NOTE: If you administered exams to students with accommodations, review the *2018-19 AP Coordinator's Manual* and the *2018-19 AP SSD Guidelines* for information about completing the Nonstandard Administration Report (NAR) form, and returning these exams.

The exam proctor should complete the following tasks if asked to do so by the AP coordinator. Otherwise, the AP coordinator must complete these tasks:

- Complete an Incident Report for any students who used extra paper for the free-response section. (Incident Report forms are provided in the coordinator packets sent with the exam shipments.) **These forms must be completed with a No. 2 pencil.** It is best to complete a single Incident Report for multiple students per exam subject, per administration (regular or late testing), as long as all required information is provided. Include all exam booklets with extra sheets of paper in an Incident Report return envelope (see page 68 of the *2018-19 AP Coordinator's Manual* for complete details).
- Before storing materials, check the "School Use Only" section on page 1 of the answer sheet and:
    - Fill in the appropriate section number circle in order to access a separate AP Instructional Planning Report (for regularly scheduled exams only) or subject score roster at the class section or teacher level. See "Post-Exam Activities" in the *2018-19 AP Coordinator's Manual*.
    - Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

Name:___________________________________

Answer Sheet for AP Physics C: Mechanics
Practice Exam, Section I

No.	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	

No.	Answer
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	
31	
32	
33	
34	
35	

AP® Physics C: Mechanics Exam

SECTION I: Multiple Choice

2019

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
45 minutes
Number of Questions
35
Percent of Total Score
50%
Writing Instrument
Pencil required
Electronic Device
Calculator allowed

Instructions

Section I of this exam contains 35 multiple-choice questions. For these questions, fill in only the circles for numbers 1 through 35 on your answer sheet. A table of information and lists of equations that may be helpful are in the booklet. Calculators, rulers and straightedges may be used in this section.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4PBP4-S
80

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\ \text{MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^{3}\ \text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \leq \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \bullet d\vec{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \bullet \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2}k(\Delta x)^2$

$x = x_{max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{Gm_1 m_2}{r^2}$

$U_G = -\frac{Gm_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \bullet d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa\varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$R = \frac{\rho\ell}{A}$

$\vec{E} = \rho\vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \bullet d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\,d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 nI$

$\Phi_B = \int \vec{B} \bullet d\vec{A}$

$\mathcal{E} = \oint \vec{E} \bullet d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

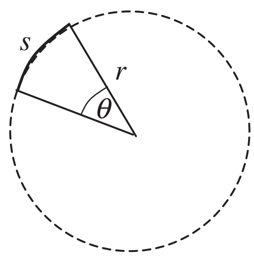
$a^2 + b^2 = c^2$

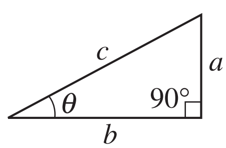
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

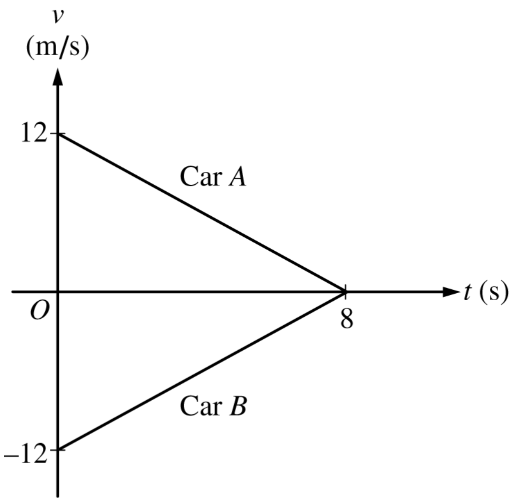
$\vec{A}\bullet\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

PHYSICS C: MECHANICS
SECTION I
Time—45 minutes
35 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding circle on the answer sheet.

Note: To simplify calculations, you may use $g = 10\,\text{m/s}^2$ in all problems.



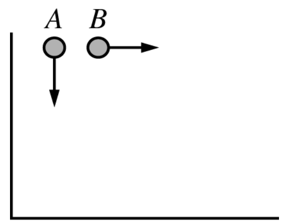
1. Cars A and B are moving in opposite directions along a straight road. They pass each other at time $t = 0$. Their velocities v are given as a function of time t in the graph above. The distance between the cars at $t = 8$ s is

   (A) Zero
   (B) 24 m
   (C) 48 m
   (D) 96 m
   (E) 192 m

GO ON TO THE NEXT PAGE.

2. A particle is moving along the y-axis. The particle's position as a function of time is given by $y = \alpha t^3 - \beta t + \phi$, where $\alpha = 1\ \text{m/s}^3$, $\beta = 4\ \text{m/s}$, and $\phi = 3\ \text{m}$. What is the particle's acceleration at time $t = 3.0\ \text{s}$?

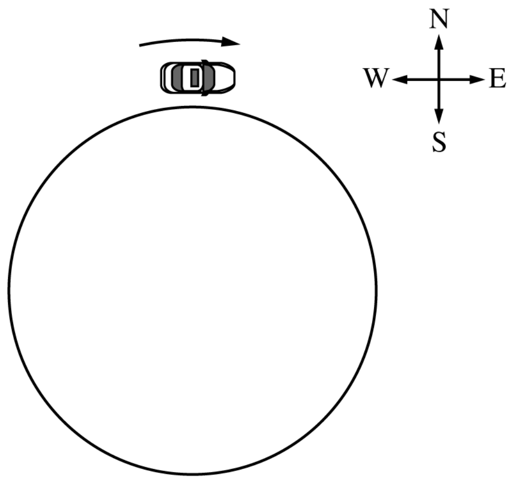
(A) $6.0\ \text{m/s}^2$
(B) $9.0\ \text{m/s}^2$
(C) $18\ \text{m/s}^2$
(D) $23\ \text{m/s}^2$
(E) $27\ \text{m/s}^2$



3. Two stones, represented in the figure above, are thrown from the same height with the same initial speed. Stone A is thrown vertically downward and stone B is thrown horizontally. If the stones are thrown at the same time and air resistance is negligible, which of the following is true?

(A) The two stones will reach the ground at the same time with the same speed.
(B) The two stones will reach the ground at the same time but with different speeds.
(C) Stone A will reach the ground first, but stone B will have the greater speed just before hitting the ground.
(D) Stone A will reach the ground first, but the two stones will have the same speed just before they hit the ground.
(E) Stone A will reach the ground first, and will have the greater speed just before hitting the ground.

Questions 4-6



Note: Figure not drawn to scale.

A car is traveling clockwise around a circular racetrack of radius 1440 m. When the car is at the northernmost point on the circle, as shown above, it has a speed of $36.0\ \text{m/s}$ and is slowing down at a rate of $1.20\ \text{m/s}^2$.

4. The direction of the velocity of the car is

   (A) due east
   (B) south of east
   (C) due south
   (D) south of west
   (E) due west

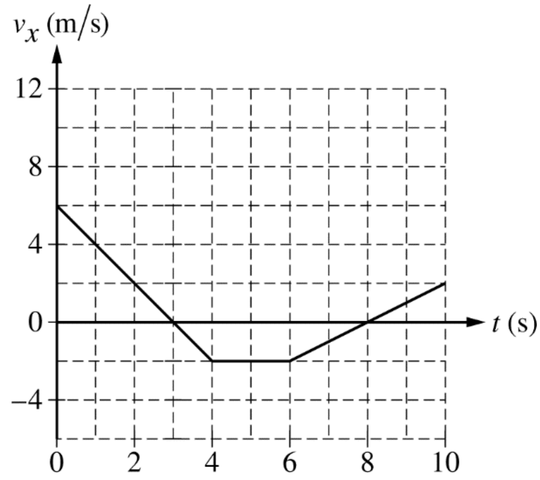
5. The direction of the acceleration of the car is

   (A) due east
   (B) south of east
   (C) due south
   (D) south of west
   (E) due west

6. What is the magnitude of the acceleration of the car?

   (A) $0.30\ \text{m/s}^2$
   (B) $0.90\ \text{m/s}^2$
   (C) $1.2\ \text{m/s}^2$
   (D) $1.5\ \text{m/s}^2$
   (E) $2.1\ \text{m/s}^2$

Questions 7-8



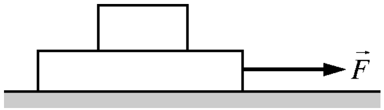
A car moves in a straight line along the x-axis. The velocity of the car v_x as a function of time t is shown in the graph above. The position x of the car at $t = 0$ is $x = 0$.

7. The average acceleration a_x of the car during the interval of 0 to 10 s is most nearly

   (A) $-2.0\ \text{m/s}^2$
   (B) $-0.40\ \text{m/s}^2$
   (C) $+0.40\ \text{m/s}^2$
   (D) $+1.0\ \text{m/s}^2$
   (E) $+2.0\ \text{m/s}^2$

8. The average velocity of the car during the interval of 0 to 10 s is most nearly

   (A) $-1.4\ \text{m/s}$
   (B) $+0.40\ \text{m/s}$
   (C) $+1.4\ \text{m/s}$
   (D) $+1.8\ \text{m/s}$
   (E) $+4.0\ \text{m/s}$



9. Two blocks rest on a table, as shown above. The bottom block is pulled to the right by an applied force $\vec{F}$ that is strong enough so that the two blocks do not move together (i.e., they do not have the same acceleration or velocity). There is friction between the blocks, but the tabletop is frictionless. When the top block leaves the bottom block, where does it land and why?

(A) The top block will land directly below where it starts because objects at rest tend to stay at rest.
(B) The top block will land to the left of where it starts because of the static friction between the blocks.
(C) The top block will land to the left of where it starts because of the kinetic friction between the blocks.
(D) The top block will land to the right of where it starts because of the static friction between the blocks.
(E) The top block will land to the right of where it starts because of the kinetic friction between the blocks.

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10. A sphere of mass m is dropped from the top of a building and reaches the ground before achieving terminal velocity. The force of air resistance that acts on the sphere as it falls is given by $F = -kv$, where k is a positive constant and v is the velocity of the sphere. What happens to the magnitude of the sphere's velocity and acceleration, and to the distance it falls during each second, as the sphere approaches the ground?

	Magnitude of Velocity	Magnitude of Acceleration	Distance of Fall Each Second
(A)	Increases	Increases	Increases
(B)	Increases	Decreases	Increases
(C)	Increases	Decreases	Decreases
(D)	Decreases	Increases	Decreases
(E)	Decreases	Decreases	Increases

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Questions 11-12

An object of mass 0.5 kg is given an initial velocity and then slides across a horizontal surface. The object experiences a resistive force that is a function of velocity. The velocity v of the object as a function of time t is given by $v(t) = \alpha e^{-\beta t}$, where $\alpha = 2\ \text{m/s}$ and $\beta = 3\ \text{s}^{-1}$.

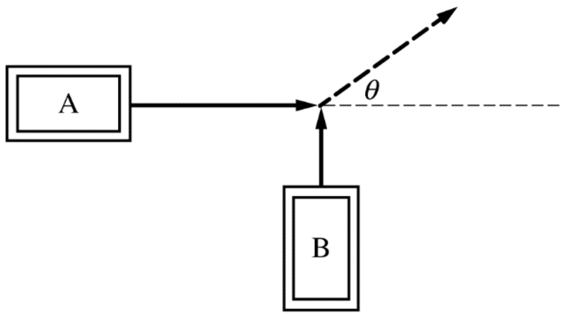
11. Which of the following is a correct expression for the net force, in newtons, exerted on the object as a function of time?

(A) $-6e^{-3t}$
(B) $-3e^{-3t}$
(C) e^{-3t}
(D) $2e^{-3t}$
(E) $\dfrac{2}{3\left(1 - e^{-3t}\right)}$

12. The energy dissipated due to the resistive force after a very long time is most nearly

(A) 0.5 J
(B) 1 J
(C) 2 J
(D) 4 J
(E) infinity

Questions 13-14



Top View

Cart A is traveling east when it collides with cart B, which is traveling north. Cart A has a mass of 3.05 kg, and cart B has a mass of 2.10 kg. The two carts travel together as a single object on a horizontal surface at an angle θ relative to due east, as shown above.

13. In one trial, the initial speed of cart A is $2.5\ \text{m/s}$ and the initial speed of cart B is $1.5\ \text{m/s}$. The angle θ relative to east that the carts travel after the collision is most nearly

(A) 22°
(B) 36°
(C) 45°
(D) 54°
(E) 62°

14. In another trial, the speed of the carts immediately after the collision is $0.60\ \text{m/s}$, and the carts slide 0.24 m on the horizontal surface before coming to rest. The coefficient of kinetic friction between the sliding carts and the surface is most nearly

(A) 0.08
(B) 0.13
(C) 0.19
(D) 0.25
(E) 0.75

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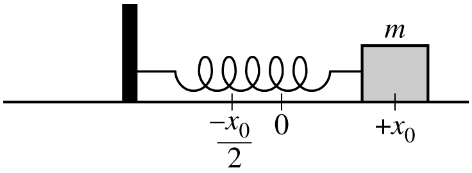
Questions 15-17



A block of mass m is on a rough horizontal surface and is attached to a spring with spring constant k. The coefficient of kinetic friction between the surface and the block is μ. When the block is at position $x = 0$, the spring is at its unstretched length. The block is pulled to position $x = +x_0$, as shown above, and released from rest. The block then travels to the left and passes through $x = 0$ before coming momentarily to rest at position $x = -x_0/2$.

15. Which of the following is a correct expression for the kinetic energy of the block as it first travels through position $x = 0$?

(A) 0

(B) $kx_0^2/2$

(C) $kx_0^2/2 - \mu mgx_0$

(D) $kx_0^2/2 - 3\mu mgx_0/2$

(E) $kx_0^2/2 - 2\mu mgx_0$

16. Which of the following is a correct expression for the coefficient of kinetic friction μ?

(A) $\dfrac{kx_0}{4mg}$

(B) $\dfrac{kx_0}{2mg}$

(C) $\dfrac{3kx_0}{4mg}$

(D) $\dfrac{kx_0}{mg}$

(E) $\dfrac{2kx_0}{mg}$

17. Which of the following is a differential equation that could be used to solve for the block's position x as a function of time t when it is moving to the left?

(A) $m\dfrac{d^2x}{dt^2} = kx + \mu mg$

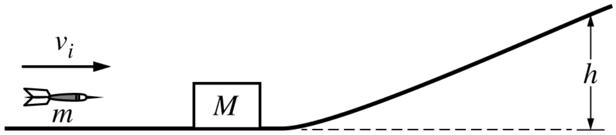
(B) $m\dfrac{d^2x}{dt^2} = kx - \mu mg$

(C) $m\dfrac{d^2x}{dt^2} = -kx + \mu mg$

(D) $m\dfrac{d^2x}{dt^2} = -kx - \mu mg$

(E) $m\dfrac{d^2x}{dt^2} = kx$

Questions 18-19



In the diagram above, a block of mass M is initially at rest on a horizontal surface at the base of an inclined plane. The surface and plane have negligible friction. The block is struck by a projectile of mass m traveling with a horizontal velocity v_i. The projectile becomes embedded in the block, and they move together to the right with speed v_f.

18. Which of the following is a correct expression for v_f?

(A) $\sqrt{gh}$

(B) $\sqrt{v_i^2 + 2gh}$

(C) $\dfrac{m}{(m + M)}v_i$

(D) $\dfrac{m}{M}v_i$

(E) $\dfrac{M}{m}v_i$

19. The block and projectile smoothly transition onto the inclined plane. Which of the following is a correct expression for the maximum height that the block moves up the inclined plane in terms of v_f?

(A) $\dfrac{v_f^2}{2g}$

(B) $\dfrac{v_f}{\sqrt{2g}}$

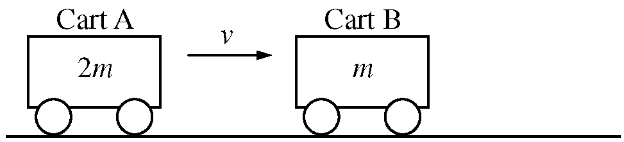
(C) $\dfrac{Mv_f^2}{2mg}$

(D) $\dfrac{2g}{v_f^2}$

(E) $\dfrac{2g}{v_f}$

20. The net force F acting on an object that moves along a straight line is given as a function of time t by $F(t) = \kappa t^2 + \tau$, where $\kappa = 1\ \text{N/s}^2$ and $\tau = 1\ \text{N}$. What is the change in momentum of the object from $t = 0$ s to $t = 3$ s?

(A) $6\ \text{kg}\bullet\text{m/s}$
(B) $10\ \text{kg}\bullet\text{m/s}$
(C) $12\ \text{kg}\bullet\text{m/s}$
(D) $30\ \text{kg}\bullet\text{m/s}$
(E) It cannot be determined without knowing the initial momentum of the object.



21. Cart A of mass $2m$ is moving with velocity v to the right on a horizontal frictionless track, as shown above, when it collides with cart B of mass m. Cart B is initially at rest, and the collision is perfectly elastic. Which of the following best describes the motion of the carts immediately after the collision?

(A) Cart A is moving to the left, and cart B is moving to the right.
(B) Cart A is moving to the left, and cart B remains stationary.
(C) Cart A is stationary, and cart B is moving to the right.
(D) Both carts move to the right, and they are stuck together.
(E) Both carts move to the right, but they are not stuck together.

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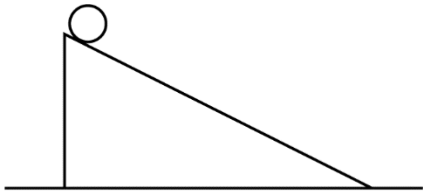
22. A ball of mass m is dropped from rest at a height h and collides elastically with the floor, rebounding to its original height. What is the magnitude of the net impulse on the ball during the collision with the floor?

(A) Zero
(B) $m\sqrt{gh}$
(C) $m\sqrt{2gh}$
(D) $m\sqrt{4gh}$
(E) $m\sqrt{8gh}$



23. A sphere starts from rest at the top of a ramp, as shown above. It rolls without slipping down the ramp. The potential energy of the sphere-Earth system is zero at the bottom of the ramp. Which of the following is true of the sphere when it reaches the bottom of the ramp?

(A) Its rotational kinetic energy equals the initial potential energy of the sphere-Earth system.
(B) Its translational kinetic energy equals the initial potential energy of the sphere-Earth system.
(C) Its translational kinetic energy and rotational kinetic energy are equal.
(D) The sum of its translational kinetic energy and rotational kinetic energy equals the initial potential energy of the sphere-Earth system.
(E) The sum of its translational kinetic energy and rotational kinetic energy equals the energy lost because of friction.

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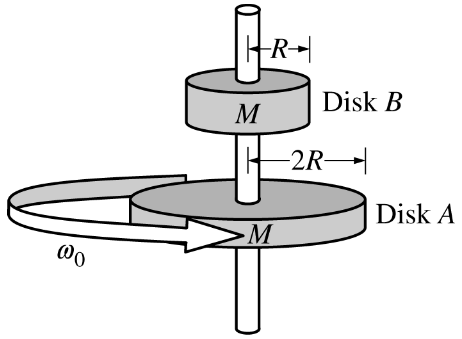

24. Two horizontal disks of mass M have the radii shown above. Disk A is attached to an axle of negligible mass spinning freely with angular velocity ω_0. Disk B, not attached to the axle and initially held at rest, is released and drops down onto disk A. When both disks spin together without slipping, the angular velocity ω_f of the disks is

(A) $\frac{1}{3}\omega_0$

(B) $\frac{1}{2}\omega_0$

(C) $\frac{2}{3}\omega_0$

(D) $\frac{4}{5}\omega_0$

(E) $\frac{2}{\sqrt{5}}\omega_0$

25. A satellite is in a circular orbit such that it stays directly over the same point on Earth's equator. Which of the following must be true for the satellite?

I. It must have a specific mass.
II. It must have a specific altitude.
III. It must have a specific angular velocity.

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) I, II, and III

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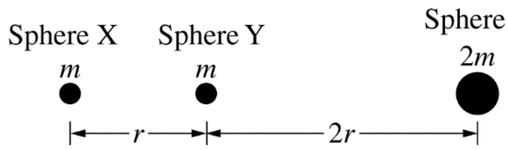


26. Spheres X, Y, and Z have the masses and locations indicated in the figure above. What is the magnitude of the net gravitational force on sphere X due to the other two spheres?

(A) $\dfrac{Gm^2}{2r^2}$

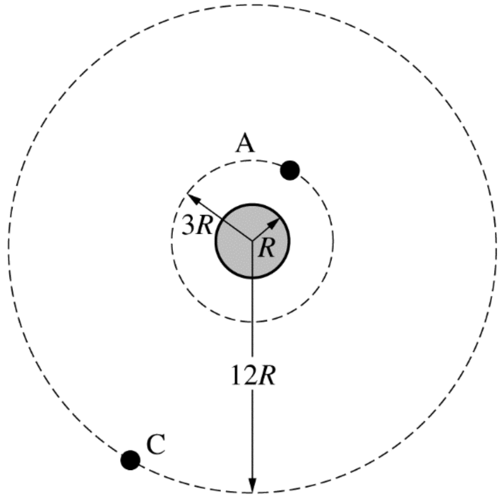
(B) $\dfrac{Gm^2}{r^2}$

(C) $\dfrac{11Gm^2}{9r^2}$

(D) $\dfrac{5Gm^2}{4r^2}$

(E) $\dfrac{2Gm^2}{r^2}$

Questions 27-28



Note: Figure not drawn to scale.

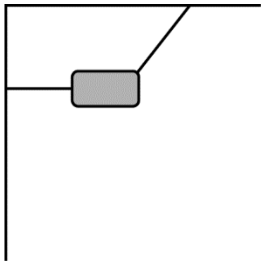
Two identical satellites orbit a planet of radius R in circular orbits A and C of radii $3R$ and $12R$, respectively, as shown above.

27. How does the magnitude of the gravitational force F_A between the planet and satellite in orbit A compare to the magnitude of the gravitational force F_C between the planet and satellite in orbit C?

(A) $F_A = 2F_C$
(B) $F_A = 3F_C$
(C) $F_A = 4F_C$
(D) $F_A = 12F_C$
(E) $F_A = 16F_C$

28. The speed of the satellite in orbit A is v_A. The speed of the satellite in orbit C is v_C. The ratio v_A/v_C is

(A) 1/2
(B) 1/1
(C) 2/1
(D) 4/1
(E) 12/1

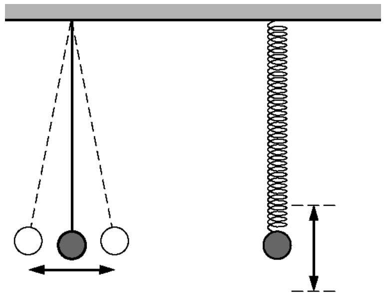


29. At the surface of Earth, an object is suspended by two cords. One cord is horizontal and one cord is at an angle to the horizontal, as shown in the figure above. The tension in the horizontal cord is T_H . If the assembly is moved to the surface of a planet with the same average density and twice the radius of Earth, the new tension in the horizontal cord will be

(A) $T_H/2$
(B) $T_H/4$
(C) T_H
(D) $2T_H$
(E) $4T_H$

30. A space shuttle has a mass of 90,000 kg. In order to stay in a circular orbit, it must have a velocity of 8000 m/s. The pilot discovers that the shuttle has slowed down to 7900 m/s, and the shuttle's speed needs to increase. If the thrusters exert a constant force of 50,000 N, how long do the thrusters have to exert this force in order to return the shuttle to orbital velocity?

(A) 30 s
(B) 60 s
(C) 120 s
(D) 180 s
(E) 240 s



31. A pendulum and a spring both have a 1 kg sphere attached to their ends and have the same length when in equilibrium, as shown in the figure above. They both oscillate with the same period. If the 1 kg spheres are replaced with 2 kg spheres and the amplitudes of oscillation are unchanged, which of the following is true about the resultant period of oscillation for each?

	Pendulum	Spring
(A)	Remains the same	Decreases
(B)	Remains the same	Increases
(C)	Remains the same	Remains the same
(D)	Increases	Remains the same
(E)	Increases	Increases

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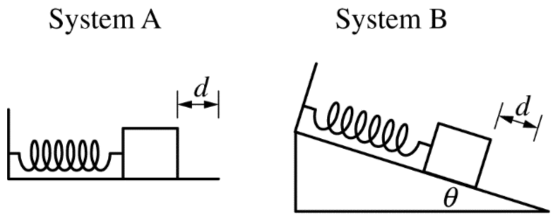
Questions 32-34



Systems A and B contain identical ideal springs and identical blocks that can slide along a surface of negligible friction. In system A, the surface is horizontal. In system B, the surface makes an angle θ with the horizontal. Initially, both blocks are at rest and in equilibrium. Each block is then pulled the same distance d in the direction shown in the figures and released from rest at $t = 0$.

32. After the block in system A is released from rest, the time for the block to first reach a maximum speed is Δt_A. After the block in system B is released, the time for the block to first reach a maximum speed is

(A) Δt_A
(B) $\Delta t_A (\sin\theta)$
(C) $\Delta t_A / (\sin\theta)$
(D) $\Delta t_A (\cos\theta)$
(E) $\Delta t_A / (\cos\theta)$

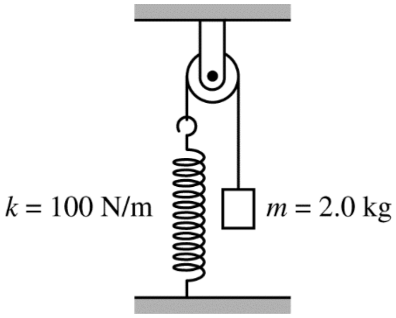
33. For system A, when the block is halfway between its release position and its equilibrium position, the block's kinetic energy is K and the elastic potential energy of the spring-block system is U. The ratio K/U is

(A) $1/3$
(B) $1/2$
(C) $1/1$
(D) $2/1$
(E) $3/1$

34. A student wants to use an apparatus similar to system B to measure the acceleration due to gravity g. If the mass of the block, the force constant of the spring, and the angle of the incline are known, what additional data must be measured to determine an experimental value for g ?

I. The stretch of the spring at the equilibrium position
II. The speed of the block as it passes the equilibrium position
III. The time interval between two consecutive passes through the equilibrium position

(A) I only
(B) II only
(C) I and II
(D) II and III
(E) I and III



35. A 2.0 kg block is attached to a string that passes over a pulley and is attached to an ideal spring of spring constant $k = 100\ \text{N/m}$, as shown above. The pulley and string have negligible mass, and there is negligible friction in the pulley. The block is held in place with the spring at its original unstretched length and then released from rest. The amplitude of the resulting oscillation is most nearly

(A) 4.0 cm
(B) 2.0 cm
(C) 10 cm
(D) 20 cm
(E) 40 cm

S T O P

END OF MECHANICS SECTION I

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON MECHANICS SECTION I ONLY.

DO NOT TURN TO ANY OTHER TEST MATERIALS.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AP® Physics C: Mechanics Exam

SECTION II: Free Response

2019

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
45 minutes
Number of Questions
3
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Electronic Device
Calculator allowed
Weight
The questions are weighted equally.

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ ☐☐ ☐☐☐☐
   Month Day Year
3. Six-digit school code
   ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. You may use any blank space in the booklet for scratch work, but you must write your answers in the spaces provided for each answer. A table of information and lists of equations that may be helpful are in the booklet. Calculators, rulers, and straightedges may be used in this section.

All final numerical answers should include appropriate units. Credit for your work depends on demonstrating that you know which physical principles would be appropriate to apply in a particular situation. Therefore, you should show your work for each part in the space provided after that part. If you need more space, be sure to clearly indicate where you continue your work. Credit will be awarded only for work that is clearly designated as the solution to a specific part of a question. Credit also depends on the quality of your solutions and explanations, so you should show your work.

Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored. You may lose credit for incorrect work that is not crossed out.

Manage your time carefully. You may proceed freely from one question to the next. You may review your responses if you finish before the end of the exam is announced.

Form I
Form Code 4PBP4-S
80

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^{3}\ \text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}{}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \bullet d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \bullet \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{\max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{Gm_1 m_2}{r^2}$

$U_G = -\frac{Gm_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \bullet d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \bullet d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \bullet d\vec{A}$

$\mathcal{E} = \oint \vec{E} \bullet d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

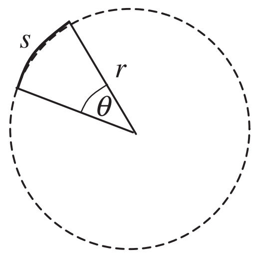
$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle



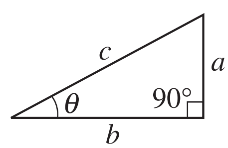
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

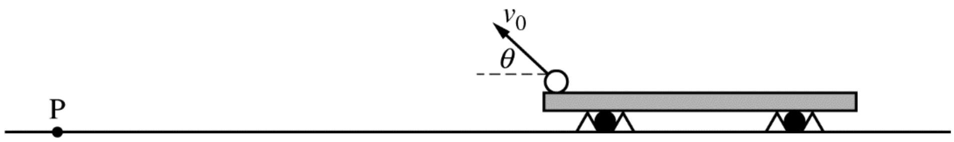
VECTOR PRODUCTS

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

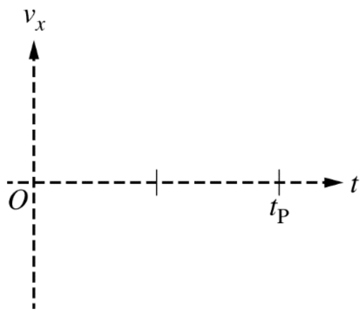


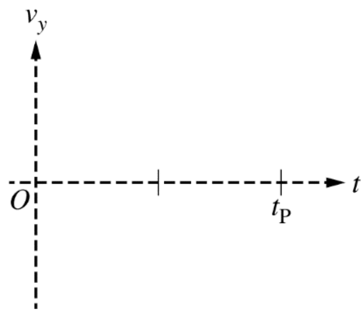
Note: Figure not drawn to scale.

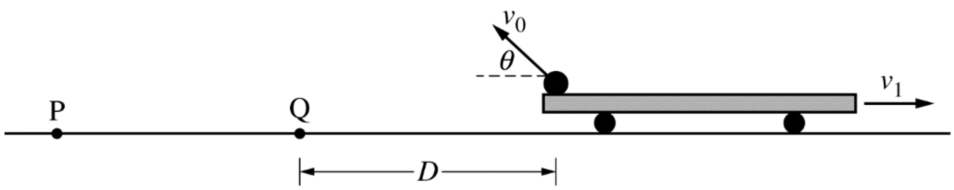
1. A projectile is launched from the back of a cart of mass *m* that is held at rest, as shown above. At time $t = 0$, the projectile leaves the cart with speed v_0 at an angle θ above the horizontal. The projectile lands at point P. Assume that the starting height of the projectile above the ground is negligible compared to the maximum height reached by the projectile and the horizontal distance traveled.

   (a) Derive an expression for the time t_P at which the projectile reaches point P. Express your answer in terms of v_0, θ, and physical constants, as appropriate.

   (b) On the axes below, sketch the horizontal component v_x and the vertical component v_y of the velocity of the projectile as a function of time *t* from $t = 0$ until $t = t_P$. Explicitly label the vertical intercepts with algebraic expressions.







Note: Figure not drawn to scale.

The projectile is again launched from the same position, but with the cart traveling to the right with speed v_1 relative to the ground, as shown above. The projectile again leaves the cart with speed v_0 relative to the cart at an angle θ above the horizontal, and the projectile lands at point Q, which is a horizontal distance D from the launching point. Express your answers in terms of v_0, θ, and physical constants, as appropriate.

(c) Give a physical reason why the projectile lands at point Q, which is not as far from the launch position as point P is, and explain how that physical reason affects the flight of the projectile.

(d) Derive an expression for v_1. Express your answer in terms of v_0, θ, D, and physical constants, as appropriate.

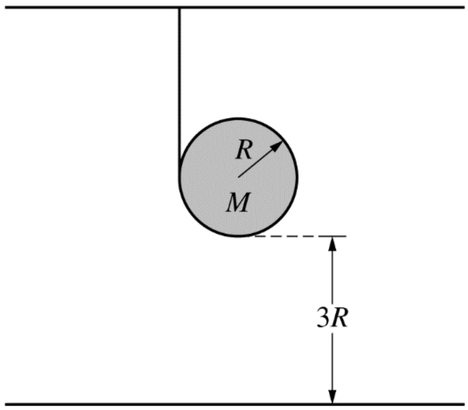
After the launch, the cart's speed is v_2. Beginning at time $t = 0$, the cart experiences a braking force of $F = -bv$, where b is a positive constant with units of kg/s and v is the speed of the cart. Express your answers to the following in terms of m, b, v_2, and physical constants, as appropriate.

(e)

i. Using Newton's second law, write but DO NOT solve a differential equation that represents the motion of the cart while it experiences the braking force.

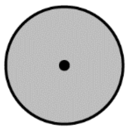
ii. Show that the speed $v(t)$ of the cart as a function of time is given by the equation $v(t) = v_2 e^{-bt/m}$.

iii. Derive an expression for the distance the cart travels from $t = 0$ until the time it comes to a stop.



2. A thin uniform disk of mass M and radius R has a string wrapped around its edge and attached to the ceiling. The bottom of the disk is at a height $3R$ above the floor, as shown above. The disk is released from rest. The rotational inertia of a disk around its center is $I = MR^2/2$.

(a) On the circle below that represents the disk, draw and label the forces (not components) that act on the disk. Each force must be represented by a distinct arrow starting on, and pointing away from, the disk, beginning at the point where the force is exerted on the disk. The dot is at the center of the disk.



(b) When released from rest, the disk falls and the string unwinds. The force the string exerts on the disk is F_T, and the gravitational force exerted on the disk is F_g. Which of the following expressions correctly relates F_T and F_g as the disk falls?

____ $F_T < F_g$ ____ $F_T = F_g$ ____ $F_T > F_g$

Justify your answer.

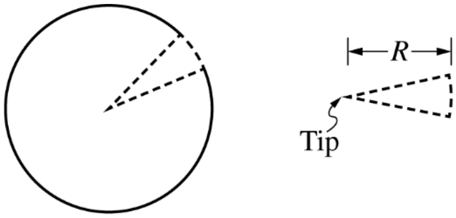
(c) Express all answers in terms of M, R, and physical constants, as appropriate.

i. Derive an expression for the acceleration a of the disk as it falls.

ii. Derive an expression for the time Δt that it takes the disk to reach the ground.

iii. Derive an expression for the rotational kinetic energy K_{rot} of the disk at the instant it reaches the ground.

Question 2 continues on the next page.



Note: Figure not drawn to scale.

(d) A very narrow wedge is cut out of the thin uniform disk of mass M, as shown above. If r is the distance from the tip of the wedge, then the linear mass density of the wedge can be expressed as follows:

$$\lambda(r) = \frac{Mr}{25R^2}.$$

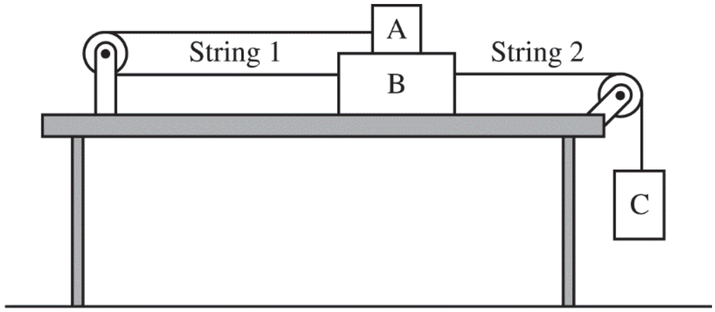
i. Using integral calculus, derive an expression for the rotational inertia of the wedge around its tip.

ii. Derive an expression for the rotational inertia of the modified disk (i.e., the disk after the narrow wedge is cut out) around its original center.

GO ON TO THE NEXT PAGE.

THIS PAGE MAY BE USED FOR SCRATCH WORK.

GO ON TO THE NEXT PAGE.



3. Three blocks are connected by strings that pass over pulleys of negligible mass. Block B is on a level, horizontal surface of negligible friction. Block A is on top of block B. String 1 connects blocks A and B. The coefficients of static and kinetic friction between blocks A and B are μ_s and μ_k, respectively. Block C is hanging over the end of the table and is attached to block B by string 2, as shown above. The masses of blocks A, B, and C are m_A, m_B, and m_C, respectively. When block C is released, the system remains at rest.

(a)

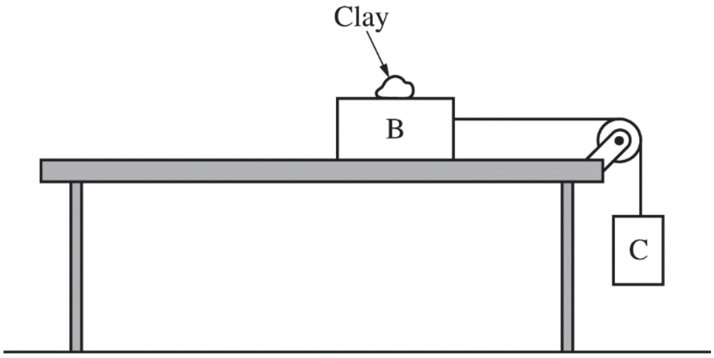
i. On the dot below, which represents block A, draw and label the forces (not components) that act on block A. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

<u>Block A</u>

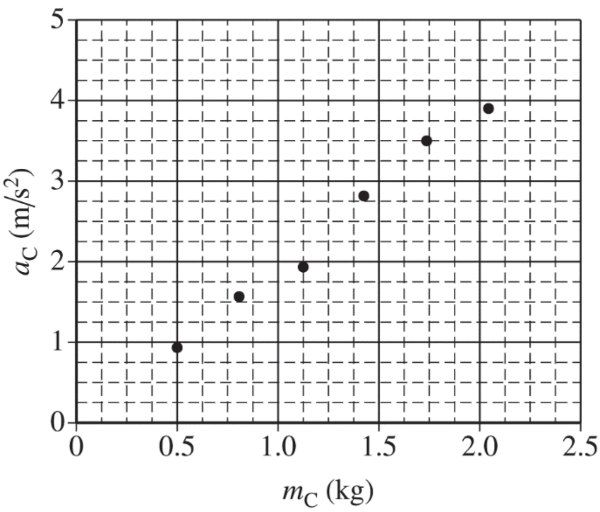
ii. On the dot below, which represents block B, draw and label the forces (not components) that act on block B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

<u>Block B</u>

(b) Derive an expression for the maximum value for m_C at which the blocks will remain at rest. Express all algebraic answers in terms of μ_s, μ_k, m_A, m_B, m_C, and physical constants, as appropriate.



The setup is modified, as shown in the figure above. Block A and one of the pulleys are removed, and block B remains on the table. There is still negligible friction between block B and the table. A lump of clay is added to block B. The students use Newton's second law to derive an equation for the acceleration a_C of block C. The acceleration is given by the equation $a_C = m_C g / m_{tot}$, where m_{tot} is the combined mass of the clay and the two blocks. Students use the setup shown above to experimentally determine the acceleration g due to gravity. In each trial, a student moves a small amount of clay from block B to block C and then releases the blocks from rest, recording the new values of m_C and a_C. The total mass of the clay and the two blocks is $m_{tot} = 5.0$ kg. The graph below shows a_C as a function of m_C, where m_C is now the combined mass of block C and the mass of clay added to block C.



(c)

i. Draw a best-fit line to the data points in the graph above.

ii. Use the best-fit line from part (c)(i) to calculate an experimental value for the acceleration g due to gravity.

(d) If the mass of the pulley in part (c) is significant, would the experimental value of g be greater than, less than, or equal to the value calculated in part (c)(ii) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

A different group of students repeats the experiment, but instead of moving clay from block B to block C, they just remove a small amount of clay from block B and set it aside, away from the setup. The equation $a_C = m_C g / m_{tot}$ still applies to the new experiment.

(e) In order to provide a straight-line graph that can be used to determine an experimental value for g, what two quantities should the students now graph? Check all that apply.

____ a_c vs $\frac{1}{m_{tot}}$ ____ a_c vs m_{tot} ____ a_c vs $\frac{m_c}{m_{tot}}$

____ a_c vs m_C ____ m_c vs m_{tot} ____ m_c vs $\frac{1}{m_{tot}}$

Justify your answer.

GO ON TO THE NEXT PAGE.

THIS PAGE MAY BE USED FOR SCRATCH WORK.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Answer Key for AP Physics C: Mechanics
Practice Exam, Section I

Question 1: D
Question 2: C
Question 3: D
Question 4: A
Question 5: D
Question 6: D
Question 7: B
Question 8: B
Question 9: E
Question 10: B
Question 11: B
Question 12: B
Question 13: A
Question 14: A
Question 15: C
Question 16: A
Question 17: C
Question 18: C
Question 19: A
Question 20: C
Question 21: E
Question 22: E
Question 23: D
Question 24: D
Question 25: D
Question 26: C
Question 27: E
Question 28: C
Question 29: D
Question 30: D
Question 31: B
Question 32: A
Question 33: E
Question 34: A
Question 35: D

Multiple-Choice Section for Physics C: Mechanics
2019 Course Framework Alignment and Rationales

Question 1

Skill	Learning Objective	Topic
4.D	CHA-1.C	Kinematics - Motion in One Dimension

(A)	Incorrect. This option uses the lines of the velocity graph meeting at $t = 8\text{s}$ to indicate that the cars are at the same location at $t = 8\text{s}$. However, the area under the curve must be used to analyze this situation.
(B)	Incorrect. This option uses the difference in the initial velocities as the distance between the cars at $t = 8\text{s}$. However, the area under the curve must be used to analyze this situation.
(C)	Incorrect. This option uses the displacement of only one of the cars as the distance between the cars at $t = 8\text{s}$. However, the area under the curve must be used to analyze this situation.
(D)	**Correct**. The area under the velocity curve is displacement; thus, at $t = 8\text{s}$, Car A has a displacement of $\text{Area} = \frac{1}{2}bh = (\frac{1}{2})(8\,\text{s})(12\,\text{m/s}) = 48\,\text{m}$ in the positive direction, and Car B has a displacement of $\text{Area} = \frac{1}{2}bh = (\frac{1}{2})(8\,\text{s})(12\,\text{m/s}) = 48\,\text{m}$ in the negative direction. Therefore, the distance between the cars is $48\,\text{m} + 48\,\text{m} = 96\,\text{m}$.
(E)	Incorrect. This option uses $d = vt$ and sets $v = 12\,\text{m/s}$ for both cars. However, the area under the curve must be used to analyze this situation.

Question 2

Skill	Learning Objective	Topic
6.C	CHA-1.B	Kinematics - Motion in One Dimension

(A)	Incorrect. This option is the second derivative of position but substitutes $t = 1\text{s}$ instead of $t = 3\text{s}$.
(B)	Incorrect. This option is the second derivative of position but substitutes $t = 1\text{s}$ instead of $t = 3\text{s}$ and adds the value of ϕ.
(C)	**Correct**. This option uses $a = \frac{d^2y}{dt^2} = \frac{d^2}{dt^2}(\alpha t^3 - \beta t + \phi) = \frac{d}{dt}(3\alpha t^2 - \beta) = 6\alpha t$. Substituting $t = 3.0\text{s}$ yields $a = 6\alpha t = (6)(1)(3) = 18\,\text{m/s}^2$.
(D)	Incorrect. This option solves for the speed, instead of the acceleration, at $t = 3\text{s}$.
(E)	Incorrect. This option solves for the speed, instead of the acceleration, at $t = 3\text{s}$. It also does not include the β term when solving for speed.

Question 3

Skill		Learning Objective	Topic
7.A		CHA-2.C	Kinematics - Motion in Two Dimensions
(A)	Incorrect. This option assumes that since the initial speeds are the same, the stones reach the ground at the same time, ignoring the different directions for the initial velocities.		
(B)	Incorrect. This option assumes that since the initial speeds are the same, the stones reach the ground at the same time, ignoring the different directions for the initial velocities. It also assumes that since stone B is not moving downward initially, it will reach the ground with a different speed.		
(C)	Incorrect. This option can be obtained by incorrectly assuming that, because t is shorter for stone A, stone B will therefore have greater speed just before hitting the ground.		
(D)	**Correct**. Both stone A and stone B initially have the same potential energy U_i and kinetic energy K_i. Both stones therefore have the same K_f at ground level, when $U_f = 0$. Substituting into conservation of energy to solve for the speeds of the stones when $$U_i + K_i = U_f + K_f$$ they reach the ground yields $mgh + \frac{1}{2}mv_0^2 = 0 + \frac{1}{2}mv_f^2$; thus, $$v_f = \sqrt{v_0^2 + 2gh}$$ because both stones have the same initial speed, they will both have the same speed when they reach the ground.		
(E)	Incorrect. This option can be obtained by incorrectly assuming that gt is the same for both rock A and rock B. For stone A, the vertical gt would be added simply to the vertical v_0, yielding $v_f = v_0 + gt$. However, for stone B, the vertical gt would be added to horizontal v_0 using vector addition, yielding $v_f = \sqrt{v_0^2 + (gt)^2}$. However, the times are not the same; thus, this analysis is not correct.		

Question 4

Skill	**Learning Objective**	**Topic**
1.D	INT-2.C	Circular Motion

(A)	**Correct**. The direction of velocity is tangent to the circle and in the direction in which the object is moving.
(B)	Incorrect. This option can be obtained by using the vector sum of the car’s velocity and the centripetal acceleration. It is not the direction of the car’s velocity.
(C)	Incorrect. This option is the direction of centripetal acceleration, not the direction of the car’s velocity.
(D)	Incorrect. This option is the vector sum of the car’s linear acceleration and centripetal acceleration. It is not the direction of the car’s velocity.
(E)	Incorrect. This option is the direction of linear acceleration, not the direction of the car’s velocity.

Question 5

Skill	**Learning Objective**	**Topic**
1.D	INT-2.C	Circular Motion

(A)	Incorrect. This option is the direction of the car’s velocity, not its acceleration.
(B)	Incorrect. This option is the vector sum of the velocity and the centripetal acceleration. It is not the car's acceleration.
(C)	Incorrect. This option is the direction of the car’s centripetal acceleration, not its acceleration.
(D)	**Correct**. The direction of acceleration is the direction of the vector sum of the linear acceleration and the centripetal acceleration.
(E)	Incorrect. This option is the direction of the car’s linear acceleration, not its acceleration.

Question 6

Skill	Learning Objective	Topic
6.B	INT-2.A	Circular Motion
(A)	Incorrect. This option is the difference between the magnitudes of the centripetal and linear accelerations. The two accelerations should be added as vectors.	
(B)	Incorrect. This option is the magnitude of the centripetal acceleration. It should be added as a vector to the car's linear acceleration.	
(C)	Incorrect. This option is the magnitude of the linear acceleration. It should be added as a vector to the car's centripetal acceleration.	
(D)	**Correct**. The magnitude of acceleration is determined by adding the linear and centripetal accelerations as vectors. The linear acceleration is $a_L = -1.20\,\text{m/s}^2$. The centripetal acceleration is given by substituting into the equation $a_C = \frac{v^2}{r} = \frac{(36.0\,\text{m/s})^2}{(1440\,\text{m})} = 0.90\,\text{m/s}^2$. Adding the two accelerations as vectors yields $a = \sqrt{a_L^2 + a_C^2} = \sqrt{(-1.20\,\text{m/s}^2)^2 + (0.90\,\text{m/s}^2)^2} = 1.50\,\text{m/s}^2$.	
(E)	Incorrect. This option adds the magnitudes of the centripetal and linear accelerations as scalars instead of as vectors.	

Question 7

Skill	Learning Objective	Topic
4.D	CHA-1.C	Kinematics - Motion in One Dimension
(A)	Incorrect. This option is the maximum acceleration, not the average acceleration.	
(B)	**Correct**. The average acceleration is the average rate of velocity change over a time interval. For a velocity-time graph, that will be the slope of the straight line connecting the starting and ending points. Solving for the slope yields $a_{\text{avg}} = \text{slope} = \frac{(2\,\text{m/s} - 6\,\text{m/s})}{(10\,\text{s} - 0\,\text{s})} = 0.40\,\text{m/s}^2$.	
(C)	Incorrect. This option calculates the correct magnitude for the average acceleration, but it has the wrong sign for the answer.	
(D)	Incorrect. This option is the final acceleration, not the average acceleration.	
(E)	Incorrect. This option is the maximum acceleration, not the average acceleration. This option also has the wrong sign.	

Question 8

Skill	**Learning Objective**	**Topic**
4.D	CHA-1.C	Kinematics - Motion in One Dimension

(A)	Incorrect. This option uses the area under the curve to determine the displacement, but it makes the area from $t = 0$ to $t = 3\text{s}$ negative instead of positive.
(B)	**Correct**. By definition, the average velocity is the average rate of position change over an interval. Mathematically, this is $\frac{\Delta x}{\Delta t}$. Therefore, using the velocity graph's Δx value, which is the sum of the signed areas under the curve, $v_{x\,\text{avg}} = \frac{\frac{1}{2}(3\,\text{s})(6\,\text{m/s})+\frac{1}{2}(1\,\text{s})(\text{-}2\,\text{m/s})+(2\,\text{s})(\text{-}2\,\text{m/s})+\frac{1}{2}(2\,\text{s})(\text{-}2\,\text{m/s})+\frac{1}{2}(2\,\text{s})(2\,\text{m/s})}{10\,\text{s}}$
(C)	Incorrect. This option uses the area under the curve to determine the displacement, but it makes the area from $t = 3\text{s}$ to $t = 8\text{s}$ positive instead of negative and the area from $t = 8\text{s}$ to $t = 10\text{s}$ negative instead of positive.
(D)	Incorrect. This option uses the area under the curve to determine the displacement, but it makes all the areas positive, and the area from $t = 3\text{s}$ to $t = 8\text{s}$ is negative.
(E)	Incorrect. This option uses the equation for constant acceleration, $\frac{1}{2}(v_0 + v_f)$, to calculate the average velocity. However, the acceleration, which is the slope of this graph, is not constant for this graph.

Question 9

Skill	Learning Objective	Topic
7.C	INT-1.E	Newton’s Laws of Motion - First and Second Laws
(A)	Incorrect. Objects at rest tend to stay at rest only when the net force exerted on them is zero.	
(B)	Incorrect. This option assumes that static friction on an object is always in the direction opposing the object’s motion. However, static friction opposes the potential motion between the two surfaces, which is not always the direction opposing the object’s motion.	
(C)	Incorrect. This option assumes that kinetic friction on an object is always in the direction opposite the object’s motion. However, kinetic friction opposes the motion between the two surfaces, which is not always the direction opposing the object’s motion.	
(D)	Incorrect. The frictional force would not even momentarily be static friction, because, the moment $\vec{F}$ is applied to the lower block, the lower block instantly accelerates. If static friction caused this acceleration, then the top block would move together with the bottom block. As described in the problem, the blocks do not move together; thus, it is not static friction between the top block and bottom block.	
(E)	**Correct**. The top block accelerates to the right, but does not move as fast as the bottom block. The force exerted on the top block is friction from the bottom block. Since the blocks do not move together, the friction is kinetic friction. Since the blocks are moving toward the right, the top block will land to the right of where it started.	

Question 10

Skill	**Learning Objective**	**Topic**
7.A	INT-1.H	Newton's Laws of Motion - First and Second Laws

(A)	Incorrect. This option shows that the magnitude of acceleration increases as the sphere gets closer to the ground. However, if the sphere is speeding up, the resistive force increases, and the net force and acceleration decrease as the stone gets closer to the ground.
(B)	**Correct**. If the sphere never reaches terminal velocity, the resistive force will always be less than the force of gravity; thus, the net force and the acceleration will always be downward. Therefore, the magnitude of the sphere's velocity will always be increasing. If the magnitude of the sphere's velocity is always increasing, the distance fallen each second will be increasing. Also, if the sphere is speeding up, the resistive force increases, and the net force and acceleration decrease as the stone gets closer to the ground.
(C)	Incorrect. This option can be obtained by not recognizing that the magnitude of velocity is the same as the distance fallen each second; thus, if the magnitude of the velocity is increasing, the distance fallen each second must also increase.
(D)	Incorrect. This option shows that the magnitude of acceleration increases as the sphere gets closer to the ground. However, if the sphere is speeding up, the resistive force increases, and the net force and acceleration decrease as the stone gets closer to the ground. It also indicates that the magnitude of the velocity and the distance fallen each second are decreasing. However, the resistive force will always be less than the force of gravity; thus, the net force and the acceleration will always be downward; thus, the magnitude of the sphere's velocity and the distance fallen each second are increasing.
(E)	Incorrect. This option indicates that the magnitude of the velocity is decreasing. However, the resistive force will always be less than the force of gravity; thus, the net force and the acceleration will always be downward; thus, the magnitude of the sphere's velocity is increasing.

Question 11

Skill		Learning Objective	Topic
6.C		INT-1.C	Newton's Laws of Motion - First and Second Laws
(A)	Incorrect. This option is the acceleration of the object, not the force.		
(B)	**Correct**. The acceleration is the derivative of the velocity, $a = \frac{dv}{dt} = \frac{d}{dt}\left(\alpha e^{-\beta t}\right) = -\beta\alpha e^{-\beta t}$. Substituting into an equation for Newton's second law yields $F_{\text{net}} = ma = m\left(-\beta\alpha e^{-\beta t}\right)$ $F_{\text{net}} = (0.5\,\text{kg})\left(-3\text{s}^{-1}\right)(2\,\text{m/s})e^{\left(-3\text{s}^{-1}\right)t} = -3e^{-3t}$.		
(C)	Incorrect. This option is the exponential part of the equation for force. It does not include the coefficient.		
(D)	Incorrect. This option is the velocity of the object, not the force.		
(E)	Incorrect. This option integrates the equation for velocity instead of taking the derivative.		

Question 12

Skill		Learning Objective	Topic
6.C		INT-4.C	Work-Energy Theorem
(A)	Incorrect. This option uses the work-energy theorem to calculate the energy dissipated but does not square the initial speed.		
(B)	**Correct**. The object will eventually come to rest; thus, the energy dissipated will be equal to initial kinetic energy. Using $t = 0$ and substituting into the kinetic energy equation yields $E = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}m\left(0 - v_i^2\right)$ $E = -\frac{1}{2}m\left(\alpha e^{-\beta t}\right)^2 = -\left(\frac{1}{2}\right)(0.50\text{kg})\left[(2\text{m/s})\left(e^0\right)\right]^2 = 1\text{J}$.		
(C)	Incorrect. This option uses the work-energy theorem to calculate the energy dissipated but does not include the mass of the object.		
(D)	Incorrect. This option uses the work-energy theorem to calculate the energy dissipated but sets the kinetic energy equal to the square of the function for speed.		
(E)	Incorrect. This option indicates that the object will move forever and then assumes the energy dissipated will be infinite.		

Question 13

Skill	**Learning Objective**	**Topic**
6.C	CON-4.F	Conservation of Linear Momentum, Collisions

(A)	**Correct**. During the collision, momentum is conserved. Momentum is a vector; thus, the horizontal and vertical directions can be analyzed independently. Substituting into conservation of momentum in the horizontal direction yields $m_A v_{Ai} = (m_A + m_B) v_{fx}$ $v_{fx} = \frac{m_A v_{Ai}}{(m_A + m_B)} = \frac{(3.05\,\text{kg})(2.5\,\text{m/s})}{(3.05\,\text{kg} + 2.10\,\text{kg})} = 1.48\,\text{m/s}$ and in the vertical direction yields $m_B v_{Bi} = (m_A + m_B) v_{fx}$ $v_{fx} = \frac{m_B v_{Bi}}{(m_A + m_B)} = \frac{(2.10\,\text{kg})(1.5\,\text{m/s})}{(3.05\,\text{kg} + 2.10\,\text{kg})} = 0.61\,\text{m/s}$. Substituting into the trigonometric function for tangent yields $0 = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{0.61\,\text{m/s}}{1.48\,\text{m/s}}\right) = 22°.$
(B)	Incorrect. This option uses the initial velocities instead of the final velocities to find the angle.
(C)	Incorrect. This option indicates that the final direction is the average of the initial directions of the two carts. However, this would only be the result if the two carts had equal initial momentums.
(D)	Incorrect. This option uses the initial velocities instead of the final velocities to find the angle. Also, it reverse the two components in the equation for tangent.
(E)	Incorrect. This option calculates the final velocity of each cart but then reverses the two components in the equation for tangent.

Question 14

Skill		Learning Objective	Topic
6.B		INT-4.C	Work-Energy Theorem
(A)	**Correct**. Setting the energy dissipated equal to the change in kinetic $$E = fd = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}m\left(0 - v_i^2\right)$$ energy yields $-\mu mgd = -\frac{1}{2}mv_i^2$. $$\mu = \frac{v_i^2}{2gd} = \frac{(0.6\,\text{m/s})^2}{2\left(9.81\,\text{m/s}^2\right)(0.24\,m)} = 0.08$$		
(B)	Incorrect. This option sets the energy dissipated equal to the change in kinetic energy but does not square the speed of the carts.		
(C)	Incorrect. This option sets the energy dissipated equal to the change in kinetic energy but does not include the one-half from the kinetic energy equation.		
(D)	Incorrect. This option sets the energy dissipated equal to the change in kinetic energy but does not square the speed of the carts. It also does not include the one-half from the kinetic energy equation.		
(E)	Incorrect. This option sets the energy dissipated equal to the change in kinetic energy but does not include g in the calculation.		

Question 15

Skill		Learning Objective	Topic
5.E		CON-2.C	Conservation of Energy
(A)	Incorrect. This option is the elastic potential energy, not the kinetic energy, at $x = 0$.		
(B)	Incorrect. This option is the kinetic energy at $x = 0$.		
(C)	**Correct**. The kinetic energy at $x = 0$ is equal to the difference of the initial elastic potential energy of the spring-block system and the energy dissipated by friction. Substituting into an energy equation $U_{Si} + fd = K_f$ yields $K_f = \frac{1}{2}kA^2 - fd = \frac{1}{2}kx_0^2 - \mu mdx_0$.		
(D)	Incorrect. This option calculates the energy dissipated by friction from the initial position to the position where the block stops, $x = -\frac{1}{2}x_0$.		
(E)	Incorrect. This option calculates the energy dissipated by friction from the initial position to a position equal to the amplitude on the negative side of equilibrium where $x = x_0$.		

Question 16

Skill	Learning Objective	Topic
5.E	CON-2.C	Conservation of Energy

(A)	**Correct**. Setting the energy dissipated by friction equal to the difference in the elastic potential energy of the spring-block system $E = fd = U_{Sf} - U_{Si} = \frac{1}{2}k\left(-\frac{1}{2}x_0\right)^2 - \frac{1}{2}k(x_0)^2$ yields $\mu mgd = \mu mg\left(\frac{3}{2}x_0\right) = \frac{3}{8}kx_0^2$. $\mu = \frac{kx_0^2}{4mg}$
(B)	Incorrect. This option substitutes into the energy equation but sets the final elastic potential energy equal to zero instead of using the elastic potential energy when the block is at $x = -\frac{1}{2}x_0$.
(C)	Incorrect. This option substitutes into the energy equation but sets the distance moved by the block equal to $\frac{1}{2}x_0$ instead of $\frac{3}{2}x_0$.
(D)	Incorrect. This option substitutes into the energy equation but sets the final elastic potential energy equal to zero instead of using the elastic potential energy when the block is at $x = -\frac{1}{2}x_0$. It also does not include the one-half in the elastic potential energy equation.
(E)	Incorrect. This option substitutes into the energy equation but sets the final elastic potential energy equal to zero instead of using the elastic potential energy when the block is at $x = -\frac{1}{2}x_0$. It also does not include the one-half in the elastic potential energy equation and includes the one-half on the energy lost side of the equation.

Question 17

Skill		Learning Objective	Topic
5.A		INT-8.D	Simple Harmonic Motion, Springs, and Pendulums
(A)	Incorrect. In this option, the spring force is in the same direction as the position of the block and is in the same direction as the friction when $x > 0$.		
(B)	Incorrect. In this option, the spring force is in the same direction as the position of the block.		
(C)	**Correct**. Substituting into a Newton's second law equation yields $F_{net} = F_S + f = ma$ $m\frac{d^2x}{dt^2} = -kx + \mu mg$.		
(D)	Incorrect. In this option, the spring force is in the same direction as the friction when $x > 0$.		
(E)	Incorrect. In this option, the effect of friction is ignored.		

Question 18

Skill		Learning Objective	Topic
5.E		CON-4.A	Conservation of Linear Momentum, Collisions
(A)	Incorrect. This option uses conservation of energy going up the hill to solve for v_f instead of using conservation of momentum for the collision. It also omits the one-half in the kinetic energy equation.		
(B)	Incorrect. This option uses the kinematics equation for going up the hill to solve for the velocity at a height h up the hill. However, v_f is the final speed for the collision, not the final speed going up the hill to a height h.		
(C)	**Correct**. Substituting into the conservation of momentum equation $p_i = p_f$ for the collision yields $mv_i = (\mathrm{m} + M)v_f$. $v_f = \frac{m}{(m + M)}v_i$		
(D)	Incorrect. This option substitutes into the conservation of momentum equation but does not include the mass of the projectile in the final momentum.		
(E)	Incorrect. This option substitutes into the conservation of momentum equation but uses the mass of the block instead of the mass of the projectile for the initial momentum. It also does not include the mass of the block in the final momentum.		

Question 19

Skill	Learning Objective	Topic
5.E	CON-2.C	Conservation of Energy

(A)	**Correct**. Substituting into the conservation of energy equation for when the block and projectile go up the hill yields $K_i + U_{gi} = K_f + U_{gf}$ $K_i + 0 = 0 + U_{gf}$ $\frac{1}{2}(m + M)v_f^2 = (m + M)gh$. $h = \frac{v_f^2}{2g}$
(B)	Incorrect. This option substitutes into the conservation of energy equation but uses h^2 for gravitational potential energy instead of h.
(C)	Incorrect. This option substitutes into the conservation of energy equation but only uses the mass of the block for the initial kinetic energy and only uses the mass of the projectile for the final potential energy.
(D)	Incorrect. This option substitutes into the conservation of energy equation but finds the reciprocal of the correct answer.
(E)	Incorrect. This option substitutes into the conservation of energy equation but finds the reciprocal of the correct answer. It also does not square the speed in the kinetic energy equation.

Question 20

Skill		Learning Objective	Topic
6.A		INT-5.B	Impulse and Momentum
(A)	Incorrect. This option takes the derivative of the force instead of integrating the equation for the force.		
(B)	Incorrect. This option substitutes into the equation for the net force instead of first integrating the equation.		
(C)	**Correct**. Integrating the equation for the net force and substituting $J = \Delta p = \int F dt = \int \left(Kt^2 + \tau\right) dt$ the given values yields $\Delta p = \int_{t=0}^{t=3} \left(t^2 + 1\right) = \left[\frac{t^3}{3} + t\right]_{t=0}^{t=3}$. $\Delta p = \left(\frac{1}{3}(3)^3 + 3\right) - 0 = 12\text{kg} \bullet \text{m/s}$		
(D)	Incorrect. This option integrates the equation for the net force, but it does not divide the terms by the new powers on the variables.		
(E)	Incorrect. The values of momentum are not needed to determine the change in momentum because the change in momentum can be determined by integrating the equation for the net force.		

Question 21

Skill		Learning Objective	Topic
7.A		CON-4.E	Conservation of Linear Momentum, Collisions
(A)	Incorrect. Only if $m_{cart\ A} < m_{cart\ B}$ would cart A move off to the left.		
(B)	Incorrect. Only if $m_{cart\ A} < m_{cart\ B}$ would cart A move off to the left. Moreover, cart B would not remain stationary. Even if $m_{cart\ A} << m_{cart\ B}$, the collision would give cart B a little velocity to the right.		
(C)	Incorrect. Only if $m_{cart\ A} = m_{cart\ B}$ would cart A be stopped by the collision.		
(D)	Incorrect. The carts would be stuck together only if the collision was inelastic, not elastic.		
(E)	**Correct**. For elastic collisions, when an object with a larger mass collides with a stationary object that has a smaller mass, both objects will move in the direction of motion of the first object. In elastic collision, the objects do not stick together; thus, the carts will move to the right and will not be stuck together.		

Question 22

Skill		Learning Objective	Topic
5.E		INT-5.B	Impulse and Momentum
(A)	Incorrect. This option indicates that the net impulse exerted on the ball is zero because the magnitude of the ball's momentum does not change during the collisions with the floor. However, this does not take into account the change in direction of the ball's momentum.		
(B)	Incorrect. This option solves for the impulse as the ball falls to the floor instead of during the collision with the floor. It also does not include the one-half in the kinetic energy equation when solving for the speed of the ball when it reaches the floor.		
(C)	Incorrect. This option solves for the impulse as the ball falls to the floor instead of during the collision with the floor.		
(D)	Incorrect. This option can result from a math error, simplifying $2m\sqrt{2gh})$ as $m\sqrt{4gh})$.		
(E)	**Correct**. The net impulse is the change in momentum of the ball during its collision with the floor. First, substituting into the conservation of energy equation to calculate the speed of the ball when it reaches the floor yields $K_i = U_{gf}$, $\frac{1}{2}mv_i^2 = mgh$, $v = \sqrt{2gh}$. The ball returns to the same height when it leaves the floor; thus, the final speed for the ball during the collision with the floor is the same as the speed when the ball reaches the floor, but it will be moving in the opposite direction. Therefore, one of the velocities must be negative. Then, substituting into the equation for change in momentum yields $\Delta p = p_f - p_i = m(v_f - v_i)$, $\Delta p = m\left(\sqrt{2gh} - \left(-\sqrt{2gh}\right)\right)$. $\Delta p = m\left(2\sqrt{2gh}\right) = m\sqrt{8gh}$		

Question 23

Skill	Learning Objective	Topic
7.A	INT-7.E	Rotational Dynamics and Energy

(A)	Incorrect. This option ignores the translational kinetic energy of the sphere.
(B)	Incorrect. This option ignores the rotational kinetic energy of the sphere.
(C)	Incorrect. While the sphere will have both translational and rotational kinetic energy at the bottom of the ramp, these two energies will not be equal.
(D)	**Correct**. The mechanical energy of the sphere-Earth system will be conserved as the sphere rolls down the incline; thus, the kinetic energy of the sphere at the bottom of the ramp must be equal to the potential energy of the sphere-Earth system when the sphere is at the top of the ramp, and the kinetic energy of the sphere is the sum of its translational kinetic energy and rotational kinetic energy.
(E)	Incorrect. There is no energy lost due to friction as the sphere rolls down the hill, because the sphere rolls without slipping.

Question 24

Skill		Learning Objective	Topic
5.A		CON-5.D	Angular Momentum and Its Conservation
(A)	Incorrect. This option substitutes into conservation of angular momentum but uses $L = mR_{total}\omega$ instead of $L = I\omega$.		
(B)	Incorrect. This option substitutes into conservation of angular momentum but uses $L = m\omega$ instead of $L = I\omega$.		
(C)	Incorrect. This option substitutes into conservation of angular momentum but uses $L = mR_{avg}\omega$ instead of $L = I\omega$.		
(D)	**Correct**. Substituting into the equation for conservation of angular momentum yields $L_i = L_f$ $I_i\omega_i = I_f\omega_f$ $I_A\omega_A = (I_A + I_B)\omega_f$ $\omega_f = \frac{I_A\omega_A}{(I_A + I_B)} = \frac{\frac{1}{2}M(2R)^2\omega_0}{\left(\frac{1}{2}M(2R)^2 + \frac{1}{2}MR^2\right)} = \frac{2MR^2\omega_0}{\left(2MR^2 + \frac{1}{2}MR^2\right)}$. $\omega_f = \frac{4}{5}\omega_0$		
(E)	Incorrect. This option uses conservation of rotational kinetic energy instead of conservation of angular momentum to solve for the final angular speed.		

Question 25

Skill	Learning Objective	Topic
7.A	CON-6.B	Orbits of Planets and Satellites

(A)	Incorrect. When setting the centripetal force on the satellite equal to the gravitational force between the satellite and Earth, the mass of the satellite cancels out of the equation; thus, the mass of the satellite is not a factor in its orbit.
(B)	Incorrect. While the satellite must be at a specific altitude, it must also have a specific angular velocity.
(C)	Incorrect. While the satellite must have a specific angular velocity, the mass of the satellite is not a factor. When setting the centripetal force on the satellite equal to the gravitational force between the satellite and Earth, the mass of the satellite cancels out of the equation.
(D)	**Correct**. Setting the centripetal force on the satellite equal to the gravitational force between the satellite and Earth and substituting into the equation yields $\begin{aligned} F_C &= F_g \\ \frac{mv^2}{r} &= G\frac{Mm}{r^2} \\ \frac{(r\omega)^2}{r} &= G\frac{M}{r^2} \\ \omega^2 &= G\frac{M}{r^3} \end{aligned}$. The mass cancels out of the equation, and the angular velocity of the satellite depends on the radius of orbit; thus, the satellite must be at a specific altitude and must also have a specific angular velocity, but the mass of the satellite is not a factor.
(E)	Incorrect. While the satellite must be at a specific altitude and must have a specific angular velocity, the mass of the satellite is not a factor. When setting the centripetal force on the satellite equal to the gravitational force between the satellite and Earth, the mass of the satellite cancels out of the equation.

Question 26

Skill	Learning Objective	Topic
5.E	FLD-1.A	Gravitational Forces

(A)	Incorrect. This option solves for the net force on sphere X but makes the distance of sphere Z as $2r$.
(B)	Incorrect. This option is the force due to only sphere Y, not the net force on sphere X.
(C)	**Correct**. Both sphere Y and Z will attract sphere X to the right. Adding the force of gravity from both spheres together and substituting into the equation for gravitational force yields $F_g = F_{gy} + F_{gz} = \frac{Gm_X m_Y}{r_{XY}^2} + \frac{Gm_X m_Z}{r_{XZ}^2} = \frac{Gm^2}{r^2} + \frac{Gm(2m)}{(3r)^2} = \frac{11Gm^2}{9r^2}$.
(D)	Incorrect. This option adds the force of gravity from spheres Y and Z together but uses $2r$ instead of $3r$ for the distance for sphere Z and uses m instead of $2m$ as the mass for Z.
(E)	Incorrect. This option adds the force of gravity from spheres Y and Z together but uses r and m for the distance and mass of both spheres.

Question 27

Skill	**Learning Objective**	**Topic**
5.B	FLD-1.A	Gravitational Forces

(A)	Incorrect. This option uses the gravitational force equation to solve for the two forces, but it uses the square root of the distance instead of the square of the distance in the equation.
(B)	Incorrect. This option uses the gravitational force equation to solve for the two forces, but it does not square the distance in the equation. It also compares the radius of orbit for satellite A to the radius of the planet instead of the radius of orbit for satellite C.
(C)	Incorrect. This option uses the gravitational force equation to solve for the two forces, but it does not square the distance in the equation.
(D)	Incorrect. This option uses the gravitational force equation to solve for the two forces, but it does not square the distance in the equation. It also uses the radius of the planet instead of the radius of orbit for the distance for satellite A.
(E)	**Correct**. Substituting into the equation for the gravitational force on the satellite in orbit A yields $F_A = \frac{Gm_S m_A}{r_A^2} = \frac{Gm_S m}{(3R)^2} = \frac{1}{9}\frac{Gm_S m}{R^2}$. Substituting into the equation for the gravitational force on the satellite in orbit C yields $F_C = \frac{Gm_S m_A}{r_A^2} = \frac{Gm_S m}{(12R)^2} = \frac{1}{144}\frac{Gm_S m}{R^2}$. $\frac{Gm_S m}{R^2} = \frac{Gm_S m}{R^2}$ Combing the two equations yields $9F_A = 144F_C$. $F_A = 16F_C$

Question 28

Skill	Learning Objective	Topic
5.B	CON-6.A	Orbits of Planets and Satellites

(A)	Incorrect. This option solves for the ratio of the speeds of satellite C to satellite A instead of the ratio of the speeds of satellite A to satellite C.
(B)	Incorrect. This option indicates that the radius of orbit does not affect the speed of the satellite. However, as the radius of orbit increases, the speed of the satellite will decrease.
(C)	**Correct**. Setting the centripetal force equal to the gravitational force and solving for the speed of satellite A yields $F_C = F_g$, $\frac{m_A v_A^2}{r_A} = \frac{GMm_A}{r_A^2}$, $v_A = \sqrt{\frac{GM}{r_A}}$. Repeating for satellite C yields $F_C = F_g$, $\frac{m_C v_C^2}{r_C} = \frac{GMm_C}{r_C^2}$, $v_C = \sqrt{\frac{GM}{r_C}}$. Taking the ratio of the two speeds yields $\frac{v_A}{v_C} = \frac{\sqrt{\frac{GM}{r_A}}}{\sqrt{\frac{GM}{r_C}}} = \sqrt{\frac{r_C}{r_A}} = \sqrt{\frac{12R}{3R}} = \frac{2}{1}$.
(D)	Incorrect. This option solves for the ratio of the square of the speeds instead of the ratio of the speeds.
(E)	Incorrect. This option indicates that the speed of the satellite is inversely proportional to the radius of orbit instead of the square root of the radius.

Question 29

Skill		Learning Objective	Topic
5.B		FLD-1.A	Gravitational Forces
(A)	Incorrect. This option solves for the tension on Earth in terms of the tension on the new planet.		
(B)	Incorrect. This option solves for the tension on the planet with the assumption that the mass of the new planet is the same with Earth's, but it has twice the radius. Thus, the weight of the object will be quarter of its value on Earth.		
(C)	Incorrect. This option indicates that the force does not depend on the planet. However, if the value of the acceleration is different on the new planet, the tension in the string will also be different.		
(D)	**Correct**. The tension in the string will depend on the weight of the block; thus, it will depend on the value of the acceleration due to gravity. Setting the weight of an object equal to the force of gravity on an object and substituting for density yields $F_W = F_g$ $mg = G\frac{Mm}{R^2}$ $g = G\frac{\rho V}{R^2} = G\frac{\rho\left(\frac{4}{3}\pi R^3\right)}{R^2} = \frac{4}{3}G\rho\pi R$; thus, if the density of the new planet is the same and the radius is twice the radius of Earth, the acceleration due to gravity and the tension in the string will both double.		
(E)	Incorrect. This option does not square the distance in the equation for tension.		

Question 30

Skill	Learning Objective	Topic
6.B	INT-5.B	Impulse and Momentum

(A)	Incorrect. This solution uses impulse-momentum to solve for time and then divides by 6 in an incorrect attempt to convert to minutes.
(B)	Incorrect. This solution sets the impulse equal to the change in momentum, but it substitutes the force for the mass and the mass for the force in the equation.
(C)	Incorrect. This solution sets the impulse equal to the change in momentum, but it substitutes the force for the mass and the mass for the force in the equation. It also includes a one-half term in the equation similar to the kinetic energy equation.
(D)	**Correct**. Setting the impulse equal to the change in momentum of the shuttle and solving for time yields $J = \Delta p$ $Ft = m(v_f - v_i)$. $t = \frac{m(v_f - v_i)}{F} = \frac{(90000\,\text{kg})(8000\text{ m/s} - 7900\text{ m/s})}{(50000\,\text{N})} = 180\,\text{s}$
(E)	Incorrect. This solution sets the impulse equal to the change in momentum, but it substitutes the force for the mass and the mass for the force in the equation. It also includes a one-fourth term in the equation similar to the squaring of the speed in the kinetic energy equation.

Question 31

Skill	**Learning Objective**	**Topic**
5.B	INT-8.K	Simple Harmonic Motion, Springs, and Pendulums

(A)	Incorrect. While the period of oscillation for the pendulum will remain the same, according to the equation for the period of oscillation of a spring is proportional to the square root of the mass. Thus, as the mass of the block increases, the period of oscillation increases. It does not decrease.
(B)	**Correct**. According to the equation for the period of a pendulum, $T_P = 2\pi\sqrt{\frac{L}{g}}$, mass is not a factor; thus, the period of oscillation will remain the same. According to the equation for the period of a spring, $T_S = 2\pi\sqrt{\frac{m}{k}}$, as the mass of the block increases, the period of oscillation increases.
(C)	Incorrect. The period of oscillation for the pendulum does not depend on mass, so it will remain the same. However, the period of oscillation of a spring is mass dependent, and it cannot not remain the same when the mass is doubled.
(D)	Incorrect. According to the equation for the period of oscillation of a pendulum, as the mass of the block increases, the period of oscillation remains the same. It does not increase. According to the equation for the period of oscillation of a spring, as the mass of the block increases, the period of oscillation increases, not decreases.
(E)	Incorrect. While the period of oscillation for the spring increases, according to the equation for the period of oscillation of a pendulum, the period of oscillation remains the same. It does not increase.

Question 32

Skill	Learning Objective	Topic
5.B	INT-8.E	Simple Harmonic Motion, Springs, and Pendulums

(A)	**Correct**. Since orientation does not affect the period of oscillation of the spring-block system, the time to reach maximum speed will be the same for system B as it was for system A.
(B)	Incorrect. This option indicates that the component of the weight of the block that is parallel to the incline will affect the acceleration of the block, causing a decrease in the period of oscillation of the spring-block system. However, the orientation of the system does not affect the period of oscillation; thus, the time to reach maximum speed is not affected by the orientation of the system.
(C)	Incorrect. This option indicates that the component of the weight of the block that is parallel to the incline will slow down the block as it moves up the incline which will increase the period of one oscillation of the spring-block system. However, the orientation of the system does not affect the period of oscillation; thus, the time to reach maximum speed is not affected by the orientation of the system.
(D)	Incorrect. This option indicates that only the component of the weight of the block that is perpendicular to the incline is significant, and it reduces the period of oscillation of the spring-block system. However, the orientation of the system does not affect the period of oscillation; thus, the time to reach maximum speed is not affected by the orientation of the system.
(E)	Incorrect. This option indicates that the component of the weight of the block that is perpendicular to the incline will increase the period of oscillation of the spring-block system. However, the orientation of the system does not affect the period of oscillation; thus, the time to reach maximum speed is not affected by the orientation of the system.

Question 33

Skill	Learning Objective	Topic
5.E	INT-8.I	Simple Harmonic Motion, Springs, and Pendulums

(A)	Incorrect. This option indicates that the ratio of the energies is 3 to 1. However, it also indicates that the potential energy is greater than the kinetic energy. Because the potential energy is proportional to the square of the position, it will have one-fourth its maximum value; thus, it will be less than, not greater than, the kinetic energy.
(B)	Incorrect. This option indicates that because the block is halfway between its release and equilibrium positions, the potential energy has twice the value of the kinetic energy. However, because the potential energy is proportional to the square of the position, it will have one-fourth its maximum value; thus, it will be less than, not greater than, the kinetic energy.
(C)	Incorrect. This option indicates that because the block is halfway between its release and equilibrium positions, both the kinetic energy and potential energy must be half their maximum value, and they would be equal to each other. However, because the potential energy is proportional to the square of the position, it will not have half the maximum value when the block is in this position.
(D)	Incorrect. This option indicates that because the block is halfway between its release and equilibrium positions, the potential energy is half its maximum value; thus, the kinetic energy has twice the value of the potential energy. However, because the potential energy is proportional to the square of the position, it will not have half the maximum value when the block is in this position.
(E)	**Correct**. The total mechanical energy of the system is the sum of the elastic potential energy of the spring-block system and the kinetic energy of the block. When the block is at its maximum displacement, the kinetic energy is zero and all the energy is elastic potential energy. Substituting into the equation for elastic potential energy yields $E = U_{S\max} = \frac{1}{2}kx_{\max}^2 == \frac{1}{2}kd^2$. Substituting into the equation for the elastic potential energy equation when the block is halfway from its maximum displacement yields $U_S = \frac{1}{2}k(\frac{1}{2}d)^2 == \frac{1}{8}kd^2$. Substituting into conservation of energy to calculate the kinetic energy of the block yields $E = U_S + K$ $\frac{1}{2}kd^2 = \frac{1}{8}kd^2 + K$; thus, the ratio of the kinetic energy to the $K = \frac{3}{8}kd^2$ elastic potential energy is $\frac{K}{U} = \frac{\frac{3}{8}kd^2}{\frac{1}{8}kd^2} = \frac{3}{1}$.

Question 34

Skill	Learning Objective	Topic
2.C	CON-1.D	Forces and Potential Energy

(A)	**Correct**. Substitution into a Newton's second law equation for the block at equilibrium in system B yields $F_{net} = 0 = F_S - W\sin\theta$; $k(\Delta x) = mg\sin\theta$; $g = \dfrac{k(\Delta x)}{m\sin\theta}$ thus, in addition to the spring constant, the mass of the block, and the angle of the incline, the stretch Δx of the spring must be known to determine a value for g.
(B)	Incorrect. Newton's second law equation for the block at equilibrium in system B yields $g = \dfrac{k(\Delta x)}{m\sin\theta}$; thus, the speed of the block is not necessary to determine a value for g.
(C)	Incorrect. Newton's second law equation for the block at equilibrium in system B yields $g = \dfrac{k(\Delta x)}{m\sin\theta}$; thus, the stretch Δx of the spring must be known, but the speed of the block is not necessary to determine a value for g.
(D)	Incorrect. Newton's second law equation for the block at equilibrium in system B yields $g = \dfrac{k(\Delta x)}{m\sin\theta}$; thus, neither the speed of the block nor the time interval between two consecutive passes through the equilibrium position is necessary to determine a value for g.
(E)	Incorrect. Newton's second law equation for the block at equilibrium in system B yields $g = \dfrac{k(\Delta x)}{m\sin\theta}$; thus, the stretch Δx of the spring must be known, but the time interval between two consecutive passes through the equilibrium position is not necessary to determine a value for g.

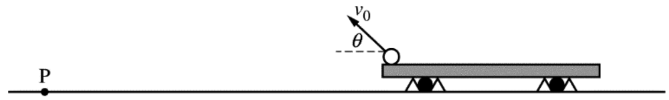
Question 35

Skill	Learning Objective	Topic
6.B	CON-1.D	Forces and Potential Energy

(A)	Incorrect. This option uses Newton’s second law to determine the amplitude of oscillation but includes the one-half term similar to the equation for the elastic potential energy. It also does not include the acceleration due to gravity in the calculation.
(B)	Incorrect. This option uses Newton’s second law to determine the amplitude of oscillation but does not include the acceleration due to gravity in the calculation.
(C)	Incorrect. This option uses Newton’s second law to determine the amplitude of oscillation but then divides the answer by two, because the block moves the same distance on both sides of the equilibrium position. However, the solution from the equation is the amplitude of oscillation.
(D)	**Correct**. The spring begins unstretched. To determine the amplitude of oscillation, Newton’s second law can be used to determine how far the block will now stretch the spring to the equilibrium position. Substituting into Newton’s second law yields $F_{net} = 0 = F_S - F_W$ $kx_{\text{max}} = mg$. $x_{\text{max}} = \frac{mg}{k} = \frac{(2.0\,\text{kg})(9.8\,\text{m/s}^2)}{(100\,\text{N/m})} = 0.20\,\text{m}$
(E)	Incorrect. This option uses Newton’s second law to determine the amplitude of oscillation but then doubles the answer, because the block moves this distance on both sides of the equilibrium position. However, the solution from the equation is the amplitude of oscillation.

Question 1

15 points total **Distribution of points**



Note: Figure not drawn to scale.

A projectile is launched from the back of a cart of mass m that is held at rest, as shown above. At time $t = 0$, the projectile leaves the cart with speed v_0 at an angle θ above the horizontal. The projectile lands at point P. Assume that the starting height of the projectile above the ground is negligible compared to the maximum height reached by the projectile and the horizontal distance traveled.

(a) LO CHA-2.C, SP 5.A, 5.E
2 points

Derive an expression for the time t_P at which the projectile reaches point P. Express your answer in terms of v_0, θ, and physical constants, as appropriate.

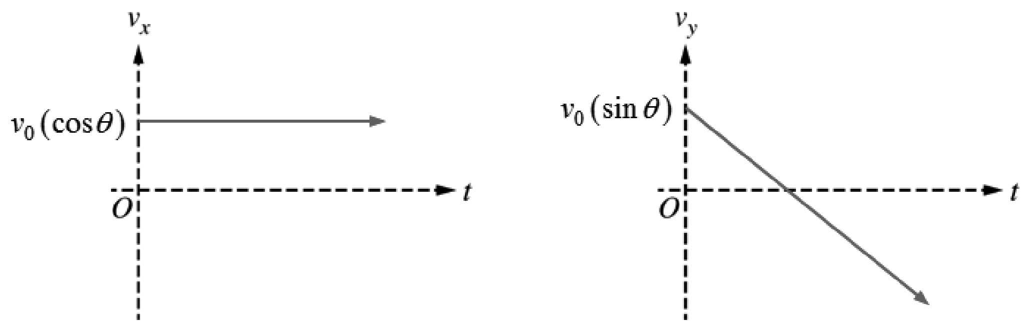
For using an appropriate kinematics equation to calculate the time to the highest point of the flight		1 point
$v_x = v_{x0} + a_x t \therefore v_y = v_{y0} + a_y t_{top}$		
For substituting into the equation above and doubling the time		1 point
$0 = v_0(\sin\theta) - g t_{top} \therefore t_{top} = \dfrac{v_0(\sin\theta)}{g}$		
$t = 2t_{top} = \dfrac{2v_0(\sin\theta)}{g}$		
Alternate solution		*Alternate Points*
For using an appropriate kinematics equation to calculate the time of flight		*1 point*
$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \therefore \Delta y = v_{1y}t + \frac{1}{2}a_y t^2$		
For substituting into the equation above		*1 point*
$0 = v_0(\sin\theta)t - \frac{1}{2}gt^2 \therefore t = \dfrac{2v_0(\sin\theta)}{g}$		

Question 1 (continued)

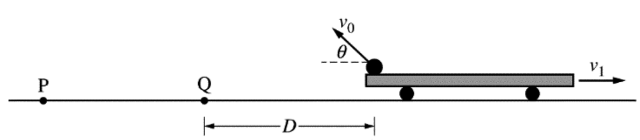
Distribution of points

(b) LO CHA-2.C, SP 3.C, 4.D
5 points

On the axes below, sketch the horizontal component v_x and the vertical component v_y of the velocity of the projectile as a function of time t from $t = 0$ until $t = t_P$. Explicitly label the vertical intercepts with algebraic expressions.



For a straight horizontal line with positive values on the v_x graph		1 point
For correctly indicating the y-intercept on the v_x graph		1 point
For a straight line with an initially positive value on the v_y graph		1 point
For a line with negative slope that crosses the horizontal axis on the v_y graph		1 point
For correctly indicating the y-intercept on the v_y graph		1 point



Note: Figure not drawn to scale.

The projectile is again launched from the same position, but with the cart traveling to the right with speed v_1 relative to the ground, as shown above. The projectile again leaves the cart with speed v_0 relative to the cart at an angle θ above the horizontal, and the projectile lands at point Q, which is a horizontal distance D from the launching point. Express your answers in terms of v_0, θ, and physical constants, as appropriate.

Question 1 (continued)

Distribution of points

(c) LO CHA-2.A.f, SP 7.D
1 point

Give a physical reason why the projectile lands at point Q, which is not as far from the launch position as point P is, and explain how that physical reason affects the flight of the projectile.

For an appropriate physical reason and correct explanation		1 point
Claim: The projectile will not travel as far as the stationary case. Evidence: The rightward component of velocity causes the initial horizontal launch velocity of the projectile with respect to the ground to be less than when the cart was stationary. Reasoning: The projectile had a component of velocity to the right with respect to the ground at the time of launch.		

(d) LO CHA-2.A.f, SP 5.A, 5.E
2 points

Derive an expression for v_1. Express your answer in terms of v_0, θ, D, and physical constants, as appropriate.

For a correct expression for the horizontal component of the velocity of the projectile		1 point
$v_x = v_0(\cos\theta) - v_1$		
For correctly substituting into the equation for constant speed		1 point
$\Delta x = v_x t \therefore D = (v_0(\cos\theta) - v_1)(2v_0(\sin\theta)/g)$		
$\dfrac{D}{2v_0(\sin\theta)/g} = v_0(\cos\theta) - v_1 \therefore v_1 = v_0(\cos\theta) - \dfrac{gD}{2v_0(\sin\theta)}$		

After the launch, the cart's speed is v_2. Beginning at time $t = 0$, the cart experiences a braking force of $F = -bv$, where b is a positive constant with units of $\mathrm{kg/s}$ and v is the speed of the cart. Express your answers to the following in terms of m, b, v_2, and physical constants, as appropriate.

Question 1 (continued)

Distribution of points

(e)

i) LO INT-1.H.a, SP 5.A
1 point

Using Newton's second law, write but DO NOT solve a differential equation that represents the motion of the cart while it experiences the braking force.

For an appropriate differential equation		1 point
$F = ma$		
$-bv = m\dfrac{dv}{dt}$		

ii) LO INT-1.H.b, SP 5.E
2 points

Show that the speed $v(t)$ of the cart as a function of time is given by the equation $v(t) = v_2 e^{-bt/m}$.

For a correct separation of variable in the above differential equation		1 point
$-\dfrac{b}{m}dt = \dfrac{1}{v}dv$		
For integrating with appropriate limits or constant of integration		1 point
$-\int_{t'=0}^{t'=t} \dfrac{b}{m}dt' = \int_{v=v_2}^{v=v(t)} \dfrac{1}{v}dv \therefore -\left[\dfrac{b}{m}t'\right]_{t'=0}^{t'=t} = \left[\ln(v)\right]_{v=v_2}^{v=v(t)}$		
$-\dfrac{b}{m}(t-0) = \ln(v(t)) - \ln(v_2) = \ln\left(\dfrac{v(t)}{v_2}\right)$		
$e^{-bt/m} = \dfrac{v(t)}{v_2} \therefore v(t) = v_2 e^{-bt/m}$		

iii) LO CHA-1.B, SP 5.E
2 points

Derive an expression for the distance the cart travels from $t = 0$ until the time it comes to a stop.

For indicating that the distance traveled is the integration of the above equation		1 point
$\Delta x = \int v dt = \int v_2 e^{-bt/m} dt$		
For integrating with appropriate limits or constant of integration		1 point
$\Delta x = \int_{t=0}^{t=\infty} v_2 e^{-bt/m} dt = v_2\left(-\dfrac{m}{b}\right)\left[e^{-bt/m}\right]_{t=0}^{t=\infty} = -\dfrac{mv_2}{b}\left(\dfrac{1}{e^{\infty}} - \dfrac{1}{e^0}\right)$		
$\Delta x = -\dfrac{mv_2}{b}(0-1) = \dfrac{mv_2}{b}$		

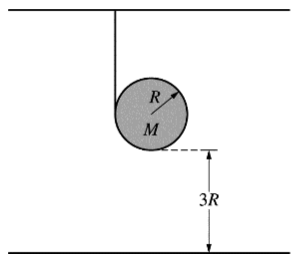
Question 1 (continued)

Learning Objectives
CHA-1.B – Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-2.A.f – Describe the velocity vector for one object relative to a second object with respect to its frame of reference.
CHA-2.C – Calculate kinematic quantities of an object in projectile motion, such as: displacement, velocity, speed, acceleration, and time, given initial conditions of various launch angles, including a horizontal launch at some point in its trajectory.
INT-1.H.a – Derive an expression for the motion of an object freely falling with a resistive drag force (or moving horizontally subject to a resistive horizontal force).
INT-1.H.b – Describe the acceleration, velocity, or position in relation to time for an object subject to a resistive force (with different initial conditions, i.e., falling from rest or projected vertically).

Science Practices
3.C – Sketch a graph that shows a functional relationship between two quantities.
4.D – Select relevant features of a graph to describe a physical situation or solve problems.
5.A – Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E – Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.D – Provide reasoning to justify a claim using physical principles or laws.

Question 2

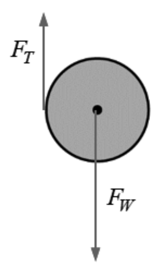
15 points total **Distribution of points**



A thin uniform disk of mass M and radius R has a string wrapped around its edge and attached to the ceiling. The bottom of the disk is at a height $3R$ above the floor, as shown above. The disk is released from rest. The rotational inertia of a disk around its center is $I = MR^2/2$.

(a) LO INT-6.B.a, SP 3.D
2 points

On the circle below that represents the disk, draw and label the forces (not components) that act on the disk. Each force must be represented by a distinct arrow starting on, and pointing away from, the disk, beginning at the point where the force is exerted on the disk. The dot is at the center of the disk.



For drawing and labeling the weight of the block directed downward starting at the center of the disk		1 point
For drawing and labeling the tension directed upward starting at the left edge of the disk		1 point
Note: A maximum of one point can be earned if there are any extraneous vectors.		

(b) LO INT-7.B.a, SP 7.A, 7.C
2 points

When released from rest, the disk falls and the string unwinds. The force the string exerts on the disk is F_T, and the gravitational force exerted on the disk is F_g. Which of the following expressions correctly relates F_T and F_g as the disk falls?

____ $F_T < F_g$ ____ $F_T = F_g$ ____ $F_T > F_g$

Justify your answer.

For selecting "$F_T < F_g$"		1 point
For a correct justification		1 point
Example Justification: The center of mass of the disk accelerates downward, so the force of gravity must be greater than the tension in the string.		

Question 2 (continued)

Distribution of points

(c)

Express all answers in terms of M, R, and physical constants, as appropriate.

i) LO INT-7.C, SP 5.A, 5.E
4 points

Derive an expression for the acceleration a of the disk as it falls.

For an expression of Newton's second law in linear form		1 point
$F_{net} = ma \therefore Mg - F_T = Ma$		
For an expression of Newton's second law in rotational form		1 point
$\tau = I\alpha \therefore F_T R = \frac{1}{2}MR^2\alpha$		
For correctly relating the linear and rotational accelerations		1 point
$F_T R = \frac{1}{2}MR^2\left(\frac{a}{R}\right) \therefore F_T = \frac{1}{2}Ma$		
For combining the two equations		1 point
$Mg - \frac{1}{2}Ma = Ma \therefore a = \frac{2}{3}g$		
Alternate Solution		*Alternate Points*
For a clear indication of using the point of contact between the string and the disk as the axis of rotation		*1 point*
For deriving an expression for the rotational inertia of the disk around an edge		*1 point*
$I = I + Mh^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$		
For an expression of Newton's second law in rotational form		*1 point*
$\tau = I\alpha \therefore MgR = \frac{3}{2}MR^2\alpha$		
For correctly relating the linear and rotational accelerations		*1 point*
$MgR = \frac{3}{2}MR^2\left(\frac{a}{R}\right) \therefore a = \frac{2}{3}g$		

Question 2 (continued)

Distribution of points

(c) con't

ii) LO CHA-1.A.b, SP 5.A, 5.E
2 points

Derive an expression for the time Δt that it takes the disk to reach the ground.

For correctly substituting the distance into a kinematic equation to calculate the time		1 point
$\Delta y = v_1 t + \frac{1}{2}at^2 \therefore 3R = 0 + \frac{1}{2}at^2$		
For substituting the acceleration from part (c)(i) into equation above		1 point
$3R = \frac{1}{3}gT^2 \therefore T = 3\sqrt{\frac{R}{g}}$		

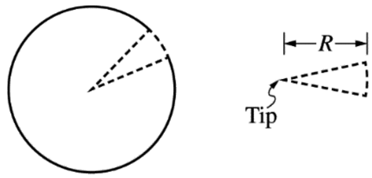
iii) LO INT-7.D.b, SP 5.E
1 point

Derive an expression for the rotational kinetic energy K_{rot} of the disk at the instant it reaches the ground.

For correctly substituting into an equation for the rotational kinetic energy of the disk		1 point
$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)(\alpha t)^2 = \left(\frac{1}{4}MR^2\right)\left(\frac{a}{R}\right)^2\left(3\sqrt{\frac{R}{g}}\right)^2$		
$K = \left(\frac{1}{4}M\right)\left(\frac{2}{3}g\right)^2\left(\frac{9R}{g}\right) = MgR$		
Alternate solution		*Alternate points*
For correctly substituting into an equation for the rotational kinetic energy of the disk		*1 point*
$mg\Delta h = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \therefore \frac{1}{2}I\omega^2 = mg\Delta h - \frac{1}{2}mv^2$		
$K = \frac{1}{2}I\omega^2 = Mg3R - \frac{1}{2}M(at)^2 = 3MgR - \frac{1}{2}M\left(\frac{2}{3}g\right)^2\left(3\sqrt{\frac{R}{g}}\right)^2 = MgR$		

Question 2 (continued)

Distribution of points



Note: Figure not drawn to scale.

(d)

A very narrow wedge is cut out of the thin uniform disk of mass M, as shown above. If r is the distance from the tip of the wedge, then the linear mass density of the wedge can be expressed as follows:

$$\lambda(r) = \frac{Mr}{25R^2}.$$

i. LO INT-6.D.c, SP 5.A, 5.E
3 points

Using integral calculus, derive an expression for the rotational inertia of the wedge around its tip.

For integrating an appropriate equation to calculate rotational inertia of the wedge		1 point
$I = \int r^2 dm$		
$m = \lambda r \therefore dm = \lambda dr = \frac{Mr}{25R^2}dr$		
For correctly substituting into equation for rotational inertia		1 point
$I = \int r^2\left(\frac{Mr}{25R^2}\right)dr \ = \frac{M}{25R^2}\int r^3 dr$		
For integrating with appropriate limits or constant of integration		1 point
$I = \frac{M}{25R^2}\int_{r=0}^{r=R} r^3 dr = \frac{M}{25R^2}\left[\frac{r^4}{4}\right]_{r=0}^{r=R} \ = \frac{M}{25R^2}\left(\frac{R^4}{4} - 0\right) = \frac{1}{100}MR^2$		

ii. LO INT-6.D.c, SP 5.E
1 point

Derive an expression for the rotational inertia of the modified disk (i.e., the disk after the narrow wedge is cut out) around its original center.

For correctly using superposition to determine the rotational inertia of the disk without the wedge		1 point
$I_{tot} = I_{disk} - I_{wedge} = \frac{1}{2}MR^2 - \frac{1}{100}MR^2 = \frac{49}{100}MR^2$		

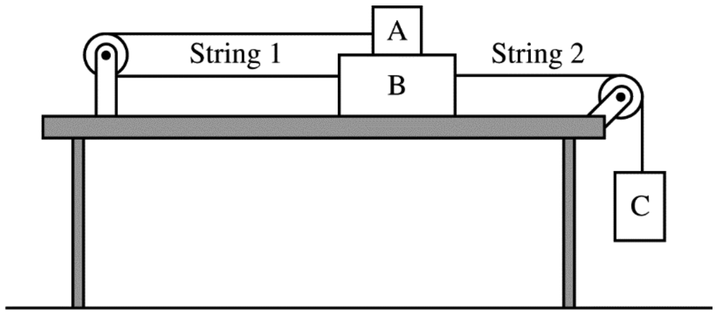
Question 2 (continued)

Learning Objectives
CHA-1.A.b – Calculate unknown variables of motion such as acceleration, velocity, or positions for an object undergoing uniformly accelerated motion in one dimension.
INT-6.B.a – Describe the two conditions of equilibrium for an extended rigid body.
INT-6.D.c – Derive the moments of inertia for a thin cylindrical shell or disc about its axis or an object that can be considered to be made up of coaxial shells (e.g., annular ring).
INT-7.B.a – Describe the net torque experienced by a rigid extended body in situations such as, but not limited to, rolling down inclines, pulled along horizontal surfaces by external forces, a pulley system (with rotational inertia), simple pendulums, physical pendulums, and rotating bars.
INT-7.C – Derive expressions for physical systems such as Atwood Machines, pulleys with rotational inertia, or strings connecting discs or strings connecting multiple pulleys that relate linear or translational motion characteristics to the angular motion characteristics of rigid bodies in the system that are: **(a)** rolling (or rotating on a fixed axis) without slipping. **(b)** rotating and sliding simultaneously.
INT-7.D.b – Calculate the total kinetic energy of a rolling body or a body that has both translation and rotational motion.

Science Practices
3.D – Create appropriate diagrams to represent physical situations.
5.A – Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E – Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A – Make a scientific claim.
7.C – Support a claim with evidence from physical representations.

Question 3

15 points total **Distribution of points**

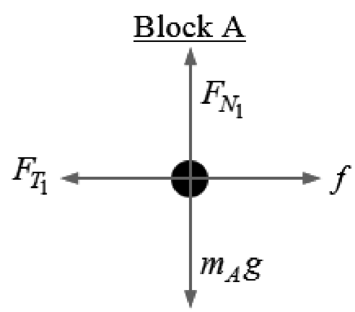


Three blocks are connected by strings that pass over pulleys of negligible mass. Block B is on a level, horizontal surface of negligible friction. Block A is on top of block B. String 1 connects blocks A and B. The coefficients of static and kinetic friction between blocks A and B are μ_s and μ_k, respectively. Block C is hanging over the end of the table and is attached to block B by string 2, as shown above. The masses of blocks A, B, and C are m_A, m_B, and m_C, respectively. When block C is released, the system remains at rest.

(a)

i) LO INT-1.A, SP 3.D
2 points

On the dot below, which represents block A, draw and label the forces (not components) that act on block A. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



For correctly drawing and labeling the vertical forces on block A		1 point
For correctly drawing and labeling the horizontal forces on block A		1 point
Note: A maximum of one point can be earned if there are any extraneous vectors		

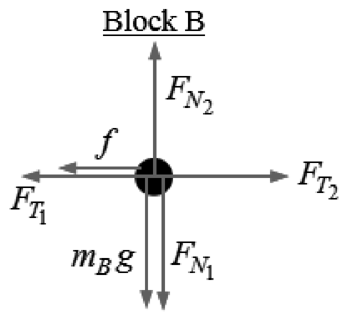
Question 3 (continued)

Distribution of points

(a) con't

ii) LO INT-1.A, SP 3.D
3 points

On the dot below, which represents block B, draw and label the forces (not components) that act on block B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



For correctly drawing and labeling the vertical forces on block B		1 point
For correctly drawing and labeling the tension in string 1 to the left and the tension in string 2 to the right on block B		1 point
For correctly drawing and labeling the static friction to the left on block B		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors		

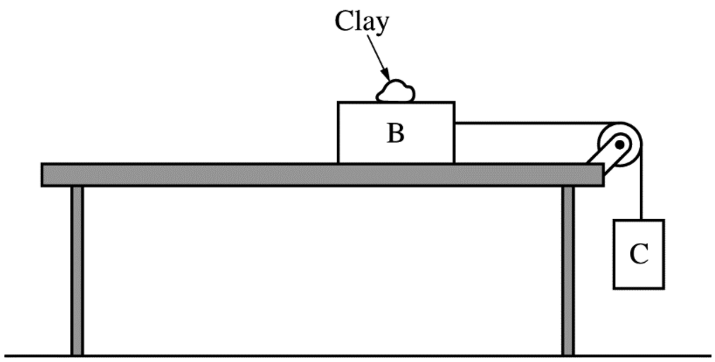
(b) LO INT-1.B.b, SP 5.A, 5.E
3 points

Derive an expression for the maximum value for m_C at which the blocks will remain at rest. Express all algebraic answers in terms of μ_s, μ_k, m_A, m_B, m_C, and physical constants, as appropriate.

For an expression of Newton's second law on block B		1 point
$F_{T_2} - f - F_{T_1} = m_B a = 0 \therefore F_{T_2} = f + F_{T_1}$		
Expression of Newton's second law on blocks A and C		
$F_{T_1} - f = m_A a = 0 \therefore F_{T_1} = f$		
$m_C g - F_{T_2} = m_C a = 0 \therefore F_{T_2} = m_C g$		
For a correct expression for the frictional force		1 point
$f = \mu_s F_N = \mu_s m_A g$		
$F_{T_2} = m_C g = f + f = 2f = 2\mu_s m_A g$		
For an answer consistent with part (a)(ii)		1 point
$m_C = 2\mu_s m_A$		

Question 3 (continued)

Distribution of points

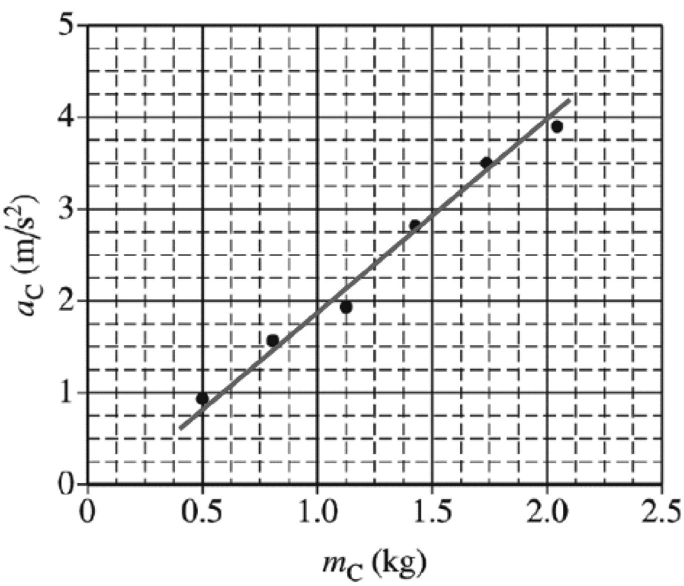


The setup is modified, as shown in the figure above. Block A and one of the pulleys are removed, and block B remains on the table. There is still negligible friction between block B and the table. A lump of clay is added to block B. The students use Newton's second law to derive an equation for the acceleration a_C of block C. The acceleration is given by the equation $a_C = m_C g / m_{tot}$, where m_{tot} is the combined mass of the clay and the two blocks. Students use the setup shown above to experimentally determine the acceleration g due to gravity. In each trial, a student moves a small amount of clay from block B to block C and then releases the blocks from rest, recording the new values of m_C and a_C. The total mass of the clay and the two blocks is $m_{tot} = 5.0$ kg. The graph below shows a_C as a function of m_C, where m_C is now the combined mass of block C and the mass of clay added to block C.

(c)

i) LO INT-1.C.e, SP 4.C
1 point

Draw a best-fit line to the data points in the graph above.



For drawing an appropriate best-fit line		1 point

Question 3 (continued)

Distribution of points

(c) con't

ii) LO INT-1.C.e, SP 4.D, 6.C
3 points

Use the best-fit line from part (c)(i) to calculate an experimental value for the acceleration g due to gravity.

For calculating the slope from the best-fit line and not from the data points unless the data points fall on the best-fit line		1 point
$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(4.0 - 1.0)\left(\text{m/s}^2\right)}{(2.05 - 0.52)(\text{kg})} = 2.0\,\text{m}/\text{kg}\bullet\text{s}^2$		
For correctly relating the slope to g		1 point
$\text{slope} = \frac{g}{m_{tot}} \therefore g = m_{tot} \times \text{slope}$		
For calculating an experimental value for g with units		1 point
$g = m_{tot} \times \text{slope} = (5.0\text{ kg})\left(2.0\,\text{m}/\text{kg}\bullet\text{s}^2\right) = 10.0\,\text{m}/\text{s}^2$		

(d) LO INT-7.A.a, SP 7.A, 7.C
1 point

If the mass of the pulley in part (c) is significant, would the experimental value of g be greater than, less than, or equal to the value calculated in part (c)(ii) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

Select "Less than"		
For a correct justification		1 point
Example Justification: If the pulley has mass, the system will have more inertia and therefore the acceleration of the system be less. If the acceleration of the system is less, the experimental value of g is less.		
Note: "Greater than" is accepted if the student provides a reasonable justification.		

A different group of students repeats the experiment, but instead of moving clay from block B to block C, they just remove a small amount of clay from block B and set it aside, away from the setup. The equation $a_C = m_C g/m_{tot}$ still applies to the new experiment.

Question 3 (continued)

Distribution of points

(e) LO INT-1.C.e, SP 7.A, 7.C
2 points

In order to provide a straight-line graph that can be used to determine an experimental value for g, what two quantities should the students now graph? Check all that apply.

____ a_c vs $\frac{1}{m_{tot}}$ ____ a_c vs m_{tot} ____ a_c vs $\frac{m_c}{m_{tot}}$

____ a_c vs m_C ____ m_c vs m_{tot} ____ m_c vs $\frac{1}{m_{tot}}$

Justify your answer.

For selecting "a_C vs $\frac{1}{m_{tot}}$" and "a_c vs $\frac{m_c}{m_{tot}}$"		1 point
For a correct justification		1 point
Example Justification: By removing the clay, the total mass is a variable. As the total mass of the system is decreased, the acceleration increase; thus a_C vs $\frac{1}{m_{tot}}$ *and* a_c vs $\frac{m_c}{m_{tot}}$ *will generate a straight line graph that can be used to determine an experimental value for g.*		

Question 3 (continued)

Distribution of points

Learning Objectives
INT-1.A – Describe an object (either in a state of equilibrium or acceleration) in different types of physical situations such as inclines, falling through air resistance, Atwood machines, or circular tracks).
INT-1.B.b – Calculate a force of unknown magnitude acting on an object in equilibrium.
INT-1.C.e – Derive a complete Newton's second law statement (in the appropriate direction) for an object in various physical dynamic situations (e.g., mass on incline, mass in elevator, strings/pulleys, or Atwood machines).
INT-7.A.a – Describe the complete analogy between fixed axis rotation and linear translation for an object subject to a net torque.

Science Practices
3.D – Create appropriate diagrams to represent physical situations.
4.C – Linearize data and/or determine a best fit line or curve.
4.D – Select relevant features of a graph to describe a physical situation or solve problems.
5.A – Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E – Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.C – Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A – Make a scientific claim.
7.C – Support a claim with evidence from physical representations.

2019 AP Physics C: Mechanics Scoring Worksheet

Section I: Multiple Choice

______________ × 1.2857 = __________________

Number Correct (out of 35) — Weighted Section I Score (Do not round)

Section II: Free Response

Question 1 ____________ (out of 15) × 1.0000 = ____________ (Do not round)

Question 2 ____________ (out of 15) × 1.0000 = ____________ (Do not round)

Question 3 ____________ (out of 15) × 1.0000 = ____________ (Do not round)

Sum = ____________

Weighted Section II Score (Do not round)

Composite Score

______________ + ______________ = ______________

Weighted Section I Score — Weighted Section II Score — Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Physics C: Mechanics

Composite Score Range	AP Score
48-90	5
36-47	4
27-35	3
21-26	2
0-20	1

2019 AP Physics C: Mechanics
Question Descriptors and Performance Data

Multiple-Choice Questions

Question	Skill	Learning Objective	Topic	Key	% Correct
1	4.D	CHA-1.C	Kinematics - Motion in One Dimension	D	74
2	6.C	CHA-1.B	Kinematics - Motion in One Dimension	C	92
3	7.A	CHA-2.C	Kinematics - Motion in Two Dimensions	D	30
4	1.D	INT-2.C	Circular Motion	A	89
5	1.D	INT-2.C	Circular Motion	D	35
6	6.B	INT-2.A	Circular Motion	D	34
7	4.D	CHA-1.C	Kinematics - Motion in One Dimension	B	83
8	4.D	CHA-1.C	Kinematics - Motion in One Dimension	B	52
9	7.C	INT-1.E	Newton’s Laws of Motion - First and Second Laws	E	48
10	7.A	INT-1.H	Newton’s Laws of Motion - First and Second Laws	B	65
11	6.C	INT-1.C	Newton’s Laws of Motion - First and Second Laws	B	71
12	6.C	INT-4.C	Work-Energy Theorem	B	34
13	6.C	CON-4.F	Conservation of Linear Momentum, Collisions	A	60
14	6.B	INT-4.C	Work-Energy Theorem	A	39
15	5.E	CON-2.C	Conservation of Energy	C	67
16	5.E	CON-2.C	Conservation of Energy	A	18
17	5.A	INT-8.D	Simple Harmonic Motion, Springs, and Pendulums	C	38
18	5.E	CON-4.A	Conservation of Linear Momentum, Collisions	C	88
19	5.E	CON-2.C	Conservation of Energy	A	77
20	6.A	INT-5.B	Impulse and Momentum	C	72
21	7.A	CON-4.E	Conservation of Linear Momentum, Collisions	E	43
22	5.E	INT-5.B	Impulse and Momentum	E	25
23	7.A	INT-7.E	Rotational Dynamics and Energy	D	82
24	5.A	CON-5.D	Angular Momentum and Its Conservation	D	37
25	7.A	CON-6.B	Orbits of Planets and Satellites	D	47
26	5.E	FLD-1.A	Gravitational Forces	C	61
27	5.B	FLD-1.A	Gravitational Forces	E	69
28	5.B	CON-6.A	Orbits of Planets and Satellites	C	25
29	5.B	FLD-1.A	Gravitational Forces	D	15
30	6.B	INT-5.B	Impulse and Momentum	D	55
31	5.B	INT-8.K	Simple Harmonic Motion, Springs, and Pendulums	B	68
32	5.B	INT-8.E	Simple Harmonic Motion, Springs, and Pendulums	A	13
33	5.E	INT-8.I	Simple Harmonic Motion, Springs, and Pendulums	E	14
34	2.C	CON-1.D	Forces and Potential Energy	A	22
35	6.B	CON-1.D	Forces and Potential Energy	D	38

2019 AP Physics C: Mechanics
Question Descriptors and Performance Data

Free-Response Questions

Question	Skill	Learning Objective	Topic	Mean Score
1	3.C|4.D|5.A|5.E|7.D	CHA-2.C|CHA-2.A.f|INT-1.H.a|INT-1.H.b|CHA-1.B	3.4	7.34
2	3.D|5.A|5.E|7.A|7.C	INT-7.C|CHA-1.A|INT-7.D|INT-6.D|INT-6.B.a|INT-7.B.a	1.3|1.4|1.5	6.4
3	3.D|4.C|4.D|5.A|5.E|6.C|7.A|7.C|7.D	INT-1.A|INT-1.B|INT-1.C	5.2	7.35